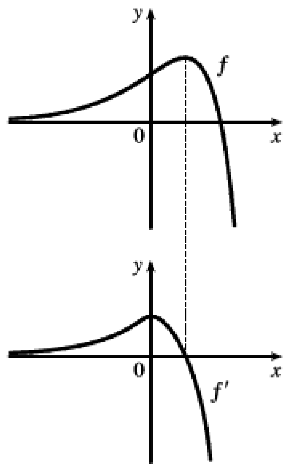
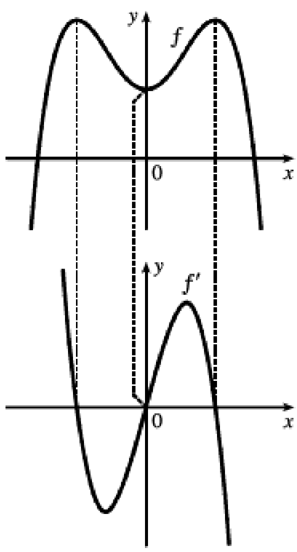
30. $2^6 = 64$, so $f(x) = x^6$ and $a = 2$.

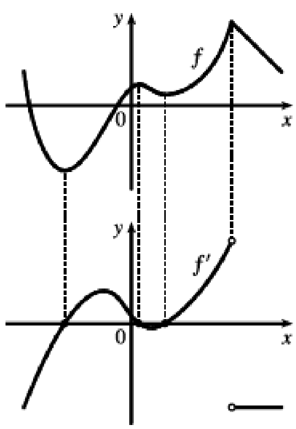
32.



33.



34.



35. (a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\sqrt{3-5(x+h)}-\sqrt{3-5x}}{h} \dfrac{\sqrt{3-5(x+h)}+\sqrt{3-5x}}{\sqrt{3-5(x+h)}+\sqrt{3-5x}}$

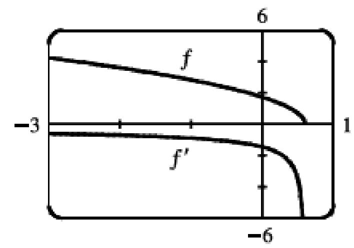
$$= \lim_{h\to 0} \frac{[3-5(x+h)]-(3-5x)}{h\left(\sqrt{3-5(x+h)}+\sqrt{3-5x}\right)} = \lim_{h\to 0} \frac{-5}{\sqrt{3-5(x+h)}+\sqrt{3-5x}} = \frac{-5}{2\sqrt{3-5x}}$$

(b) Domain of f: (the radicand must be nonnegative) $3-5x \geq 0 \quad \Rightarrow$

$5x \leq 3 \quad \Rightarrow \quad x \in \left(-\infty, \frac{3}{5}\right]$

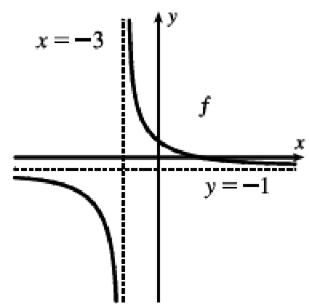
Domain of f': exclude $\frac{3}{5}$ because it makes the denominator zero;

$x \in \left(-\infty, \frac{3}{5}\right)$

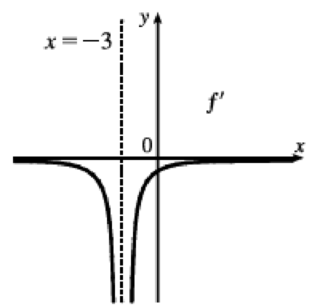


(c) Our answer to part (a) is reasonable because $f'(x)$ is always negative and f is always decreasing.

36. (a) As $x \to \pm\infty$, $f(x) = (4-x)/(3+x) \to -1$, so there is a horizontal asymptote at $y=-1$. As $x \to -3^+$, $f(x) \to \infty$, and as $x \to -3^-$, $f(x) \to -\infty$. Thus, there is a vertical asymptote at $x=-3$.



(b) Note that f is decreasing on $(-\infty, -3)$ and $(-3, \infty)$, so f' is negative on those intervals. As $x \to \pm\infty$, $f' \to 0$. As $x \to -3^-$ and as $x \to -3^+$, $f' \to -\infty$.



(c) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim\limits_{h\to 0} \dfrac{\dfrac{4-(x+h)}{3+(x+h)} - \dfrac{4-x}{3+x}}{h}$

$$= \lim_{h\to 0} \frac{(3+x)\,[4-(x+h)] - (4-x)\,[3+(x+h)]}{h\,[3+(x+h)]\,(3+x)}$$

$$= \lim_{h\to 0} \frac{(12-3x-3h+4x-x^2-hx)-(12+4x+4h-3x-x^2-hx)}{h[3+(x+h)](3+x)}$$

$$= \lim_{h\to 0} \frac{-7h}{h\,[3+(x+h)]\,(3+x)} = \lim_{h\to 0} \frac{-7}{[3+(x+h)]\,(3+x)} = -\frac{7}{(3+x)^2}$$

(d) The graphing device confirms our graph in part (b).

37. f is not differentiable: at $x=-4$ because f is not continuous, at $x=-1$ because f has a corner, at $x=2$ because f is not continuous, and at $x=5$ because f has a vertical tangent.

38. The graph of a has tangent lines with positive slope for $x < 0$ and negative slope for $x > 0$, and the values of c fit this pattern, so c must be the graph of the derivative of the function for a. The graph of c has horizontal tangent lines to the left and right of the x-axis and b has zeros at these points. Hence, b is the graph of the derivative of the function for c. Therefore, a is the graph of f, c is the graph of f', and b is the graph of f''.

39. $C'(1990)$ is the rate at which the total value of U.S. currency in circulation is changing in billions of dollars per year. To estimate the value of $C'(1990)$, we will average the difference quotients obtained using the times $t = 1985$ and $t = 1995$.

Let $A = \dfrac{C(1985) - C(1990)}{1985 - 1990} = \dfrac{187.3 - 271.9}{-5} = \dfrac{-84.6}{-5} = 16.92$ and

$B = \dfrac{C(1995) - C(1990)}{1995 - 1990} = \dfrac{409.3 - 271.9}{5} = \dfrac{137.4}{5} = 27.48$. Then

$C'(1990) = \displaystyle\lim_{t \to 1990} \frac{C(t) - C(1990)}{t - 1990} \approx \frac{A + B}{2} = \frac{16.92 + 27.48}{2} = \frac{44.4}{2} = 22.2$ billion dollars/year.

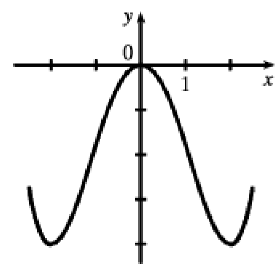
40. Let $C(t)$ be the function that denotes the cost of living in terms of time t. $C(t)$ is an increasing function, so $C'(t) > 0$. Since the cost of living is rising at a slower rate, the slopes of the tangent lines are positive but decreasing as t increases. Hence, $C''(t) < 0$.

41. (a) $f'(x) > 0$ on $(-2, 0)$ and $(2, \infty)$ $\Rightarrow$ f is increasing on those intervals. $f'(x) < 0$ on $(-\infty, -2)$ and $(0, 2)$ $\Rightarrow$ f is decreasing on those intervals.

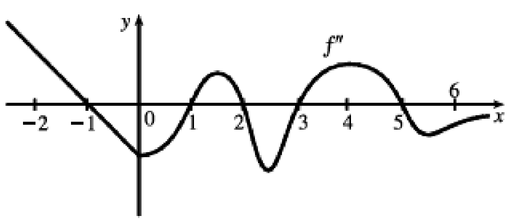
(b) $f'(x) = 0$ at $x = -2$, 0, and 2, so these are where local maxima or minima will occur. At $x = \pm 2$, f' changes from negative to positive, so f has local minima at those values. At $x = 0$, f' changes from positive to negative, so f has a local maximum there.

(c) f' is increasing on $(-\infty, -1)$ and $(1, \infty)$ $\Rightarrow$ $f'' > 0$ and f is concave upward on those intervals. f' is decreasing on $(-1, 1)$ $\Rightarrow$ $f'' < 0$ and f is concave downward on this interval.

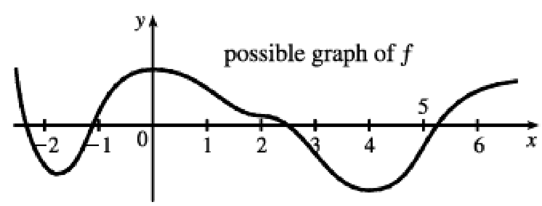
(d)



42. (a)



(b)



43. $f(0) = 0$, $f'(-2) = f'(1) = f'(9) = 0$, $\displaystyle\lim_{x \to \infty} f(x) = 0$, $\displaystyle\lim_{x \to 6} f(x) = -\infty$,

$f'(x) < 0$ on $(-\infty, -2)$, $(1, 6)$, and $(9, \infty)$, $f'(x) > 0$ on $(-2, 1)$ and $(6, 9)$,

$f''(x) > 0$ on $(-\infty, 0)$ and $(12, \infty)$, $f''(x) < 0$ on $(0, 6)$ and $(6, 12)$