1. (a) (i) $\lim_{x\to 2^+} f(x) = 3$ (ii) $\lim_{x\to -3^+} f(x) = 0$

(iii) $\lim_{x\to -3} f(x)$ does not exist since the left and right limits are not equal. (The left limit is -2.)

(iv) $\lim_{x\to 4} f(x) = 2$

(v) $\lim_{x\to 0} f(x) = \infty$ (vi) $\lim_{x\to 2^-} f(x) = -\infty$

(vii) $\lim_{x\to \infty} f(x) = 4$ (viii) $\lim_{x\to -\infty} f(x) = -1$

(b) The equations of the horizontal asymptotes are $y = -1$ and $y = 4$.

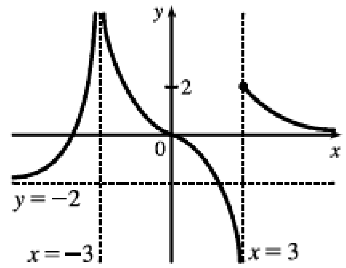
(c) The equations of the vertical asymptotes are $x = 0$ and $x = 2$.

(d) f is discontinuous at $x = -3$, 0, 2, and 4. The discontinuities are jump, infinite, infinite, and removable, respectively.

2. $\lim_{x\to -\infty} f(x) = -2$, $\lim_{x\to \infty} f(x) = 0$, $\lim_{x\to -3} f(x) = \infty$,

$\lim_{x\to 3^-} f(x) = -\infty$, $\lim_{x\to 3^+} f(x) = 2$,

f is continuous from the right at 3



3. Since the exponential function is continuous, $\lim_{x\to 1} e^{x^3 - x} = e^{1-1} = e^0 = 1$.

4. Since rational functions are continuous, $\lim_{x\to 3} \dfrac{x^2-9}{x^2+2x-3} = \dfrac{3^2-9}{3^2+2(3)-3} = \dfrac{0}{12} = 0$.

5. $$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3} = \lim_{x\to -3} \frac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x\to -3} \frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$$

6. $\lim_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3} = -\infty$ since $x^2+2x-3 \to 0$ as $x \to 1^+$ and $\dfrac{x^2-9}{x^2+2x-3} < 0$ for $1 < x < 3$.

7. $$\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \lim_{h\to 0} \frac{(h^3-3h^2+3h-1)+1}{h} = \lim_{h\to 0} \frac{h^3-3h^2+3h}{h} = \lim_{h\to 0} \left(h^2-3h+3\right) = 3$$

Another solution: Factor the numerator as a sum of two cubes and then simplify.

$$\lim_{h\to 0} \frac{(h-1)^3+1}{h} = \lim_{h\to 0} \frac{(h-1)^3+1^3}{h} = \lim_{h\to 0} \frac{[(h-1)+1]\left[(h-1)^2-1(h-1)+1^2\right]}{h}$$
$$= \lim_{h\to 0} \left[(h-1)^2-h+2\right] = 1-0+2 = 3$$

8. $$\lim_{t\to 2} \frac{t^2-4}{t^3-8} = \lim_{t\to 2} \frac{(t+2)(t-2)}{(t-2)(t^2+2t+4)} = \lim_{t\to 2} \frac{t+2}{t^2+2t+4} = \frac{2+2}{4+4+4} = \frac{4}{12} = \frac{1}{3}$$

9. $\lim_{r\to 9} \dfrac{\sqrt{r}}{(r-9)^4} = \infty$ since $(r-9)^4 \to 0$ as $r \to 9$ and $\dfrac{\sqrt{r}}{(r-9)^4} > 0$ for $r \neq 9$.

10. $$\lim_{v\to 4^+} \frac{4-v}{|4-v|} = \lim_{v\to 4^+} \frac{4-v}{-(4-v)} = \lim_{v\to 4^+} \frac{1}{-1} = -1$$

11. Let $t = \sin x$. Then as $x \to \pi^-$, $\sin x \to 0^+$, so $t \to 0^+$. Thus, $\lim\limits_{x\to\pi^-} \ln(\sin x) = \lim\limits_{t\to 0^+} \ln t = -\infty$.

12. $$\lim_{x\to-\infty} \frac{1-2x^2-x^4}{5+x-3x^4} = \lim_{x\to-\infty} \frac{(1-2x^2-x^4)/x^4}{(5+x-3x^4)/x^4} = \lim_{x\to-\infty} \frac{1/x^4-2/x^2-1}{5/x^4+1/x^3-3} = \frac{0-0-1}{0+0-3} = \frac{-1}{-3} = \frac{1}{3}$$

13. $$\begin{aligned}\lim_{x\to\infty}\left(\sqrt{x^2+4x+1}-x\right) &= \lim_{x\to\infty}\left[\frac{\sqrt{x^2+4x+1}-x}{1}\cdot\frac{\sqrt{x^2+4x+1}+x}{\sqrt{x^2+4x+1}+x}\right]\\ &= \lim_{x\to\infty}\frac{(x^2+4x+1)-x^2}{\sqrt{x^2+4x+1}+x}\\ &= \lim_{x\to\infty}\frac{(4x+1)/x}{(\sqrt{x^2+4x+1}+x)/x} \qquad \left[\text{divide by } x=\sqrt{x^2} \text{ for } x>0\right]\\ &= \lim_{x\to\infty}\frac{4+1/x}{\sqrt{1+4/x+1/x^2}+1} = \frac{4+0}{\sqrt{1+0+0}+1} = \frac{4}{2} = 2\end{aligned}$$

14. Let $t = x - x^2 = x(1-x)$. Then as $x \to \infty$, $t \to -\infty$, and $\lim\limits_{x\to\infty} e^{x-x^2} = \lim\limits_{t\to-\infty} e^t = 0$.

15. Let $t = 1/x$. Then as $x \to 0^+$, $t \to \infty$, and $\lim\limits_{x\to 0^+} \tan^{-1}(1/x) = \lim\limits_{t\to\infty} \tan^{-1} t = \dfrac{\pi}{2}$.

16. $$\begin{aligned}\lim_{x\to 1}\left(\frac{1}{x-1}+\frac{1}{x^2-3x+2}\right) &= \lim_{x\to 1}\left[\frac{1}{x-1}+\frac{1}{(x-1)(x-2)}\right] = \lim_{x\to 1}\left[\frac{x-2}{(x-1)(x-2)}+\frac{1}{(x-1)(x-2)}\right]\\ &= \lim_{x\to 1}\left[\frac{x-1}{(x-1)(x-2)}\right] = \lim_{x\to 1}\frac{1}{x-2} = \frac{1}{1-2} = -1\end{aligned}$$

19. Since $2x-1 \le f(x) \le x^2$ for $0<x<3$ and $\lim\limits_{x\to 1}(2x-1) = 1 = \lim\limits_{x\to 1} x^2$, we have $\lim\limits_{x\to 1} f(x) = 1$ by the Squeeze Theorem.

20. Let $f(x) = -x^2$, $g(x) = x^2\cos(1/x^2)$ and $h(x) = x^2$. Then since $\left|\cos(1/x^2)\right| \le 1$ for $x \ne 0$, we have $f(x) \le g(x) \le h(x)$ for $x \ne 0$, and so $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} h(x) = 0 \;\Rightarrow\; \lim\limits_{x\to 0} g(x) = 0$ by the Squeeze Theorem.

21. (a) $f(x) = \sqrt{-x}$ if $x<0$, $f(x) = 3-x$ if $0 \le x < 3$, $f(x) = (x-3)^2$ if $x>3$.

(i) $\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+}(3-x) = 3$

(ii) $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-}\sqrt{-x} = 0$

(iii) Because of (i) and (ii), $\lim\limits_{x\to 0} f(x)$ does not exist.

(iv) $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-}(3-x) = 0$

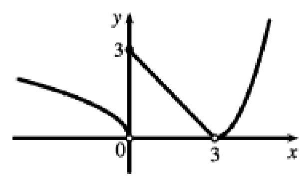
(v) $\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+}(x-3)^2 = 0$

(vi) Because of (iv) and (v), $\lim\limits_{x\to 3} f(x) = 0$.

(b) f is discontinuous at 0 since $\lim\limits_{x\to 0} f(x)$ does not exist.

f is discontinuous at 3 since $f(3)$ does not exist.

(c)



25. (a) $s = s(t) = 1 + 2t + t^2/4$. The average velocity over the time interval $[1, 1+h]$ is

$$v_{\text{ave}} = \frac{s(1+h) - s(1)}{(1+h) - 1} = \frac{1 + 2(1+h) + (1+h)^2/4 - 13/4}{h} = \frac{10h + h^2}{4h} = \frac{10+h}{4}.$$

So for the following intervals the average velocities are:

(i) $[1, 3]$: $h = 2$, $v_{\text{ave}} = (10+2)/4 = 3$ m/s

(ii) $[1, 2]$: $h = 1$, $v_{\text{ave}} = (10+1)/4 = 2.75$ m/s

(iii) $[1, 1.5]$: $h = 0.5$, $v_{\text{ave}} = (10+0.5)/4 = 2.625$ m/s

(iv) $[1, 1.1]$: $h = 0.1$, $v_{\text{ave}} = (10+0.1)/4 = 2.525$ m/s

(b) When $t = 1$, the instantaneous velocity is $\lim_{h \to 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \to 0} \frac{10+h}{4} = \frac{10}{4} = 2.5$ m/s.