Math 560

Rates of Change: Position, Velocity, Acceleration

§3.3

The position of a particle is given by the equation

$$s(t) = t^3 - 10t^2 + 25t \qquad \text{for} \qquad 0 \le t \le 6$$

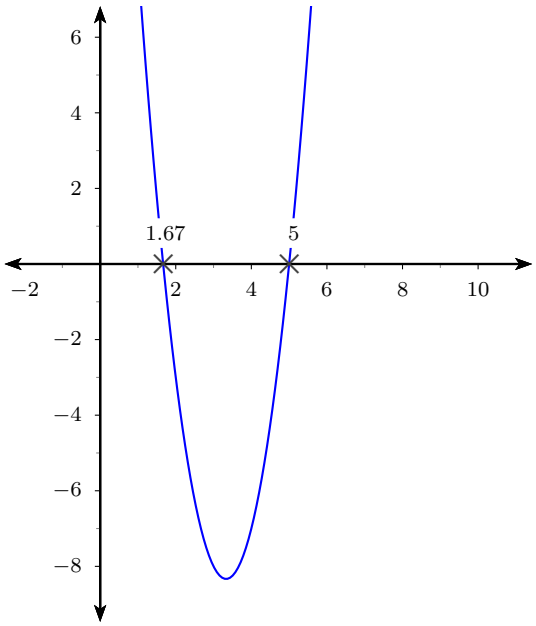
where t is measured in seconds and s in meters.

1. Determine the velocity of the particle at any time t.

Solution: $v(t) = s'(t) = 3t^2 - 20t + 25$

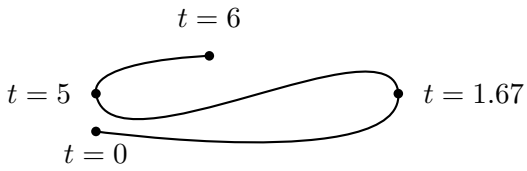
2. Determine the time(s) t when the particle is moving (a) forward, (b) backward, (c) and at rest.

Solution: (a) the particle is moving forward when $v(t) > 0$.
(b) the particle is moving forward when $v(t) < 0$.
(c) the particle is at rest when $v(t) = 0$.



From the graph of $v(t)$ we see that the particle is moving,
forward when $0 < t < 1.67$ and $5 < t < 6$
backward when $1.67 < t < 5$
rest when $t = 1.67$ and $t = 5$.

3. Draw a diagram to represent the motion of the particle.



Solution:

4. Determine the total distance traveled by the particle during the first 6 seconds.

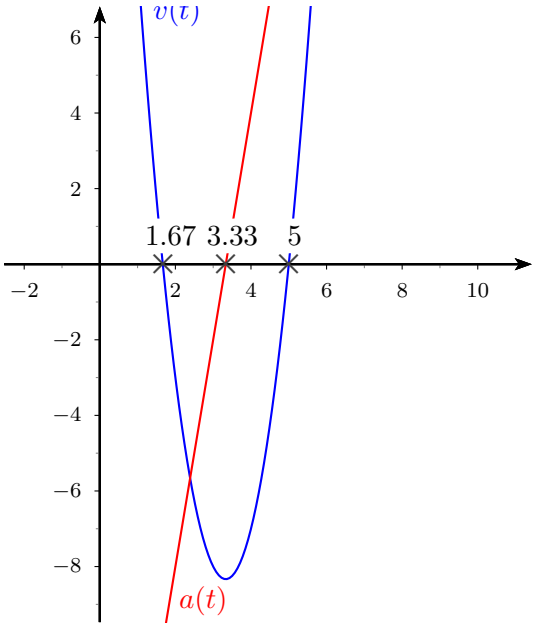
Solution: $|s(1.67) - s(0)| + |s(1.67) - s(5)| + |s(6) - s(5)| = 18.52 + 18.52 + 16 = 43.04$

5. Determine the acceleration of the particle at any time t.

Solution: $a(t) = v'(t) = 6t - 20$

6. Sketch the graph of the velocity and acceleration functions on the same axis.

Solution:



7. Determine the values of t when the particle is (a) speeding up and (b) slowing down.

Solution: (a) speeding up when $v(t)$ and $a(t)$ have the same sign. So for $1.67 < t < 3.33$ and $5 < t < 6$.
(b) slowing down when $v(t)$ and $a(t)$ have opposite signs. So for $0 < t < 1.67$ and $3.33 < t < 5$.