Math 560

Test 2 Review

§§*2.4-2.5*

1. Use the function below to answer the following questions.

$$f(x) = \begin{cases} x^2 - 1 & \text{when } -1 \leq x < 0 \\ 2x & \text{when } 0 < x < 1 \\ 1 & \text{when } x = 0 \\ -2x + 4 & \text{when } 1 < x < 2 \\ 0 & \text{when } 2 < x < 3 \end{cases}$$

(a) Determine the left and right hand limits of f at $x = -1,\ 0,\ 1,\ 2,$ and 3.

Solution:

x	left-hand limit	right-hand limit
-1	DNE	0
0	-1	0
1	2	2
2	0	0
3	0	DNE

(b) Discuss the continuity of f. Identify the types of discontinuities that exist, if any.

Solution: Not continuous at $x = -1$ and 3 because $\lim_{x\to -1} f(x) = \lim_{x \to 3} f(x)$ DNE because f is not defined for $x < -1$ and $3 < x$. Also, f is not continuous at $x = 0$ because $\lim_{x\to 0} f(x)$ DNE. Also, f is not continuous at $x = 1$ and 2 because $f(1)$ and $f(2)$ are not defined.

(c) What values of $f(1)$ and $f(2)$ should you assign to make $f(x)$ continuous at $x = 1$ and $x = 2$?

Solution: $f(1) = 2$ and $f(2) = 0$

(d) Is is possible to make f continuous at $x = 0$? If so, how?

Solution: No because it is a jump discontinuity.

(e) Is is possible to make f continuous at $x = 3$? If so, how?

Solution: Yes. Define $f(x) = 0$ for $3 \leq x$.

2. Sketch a graph of a function with all the following characteristics.

$$\lim_{x\to-\infty} f(x) = -2 \qquad \lim_{x\to-1^-} f(x) = +\infty \qquad \lim_{x\to-1^+} f(x) = +\infty \qquad f(-1) = 0$$

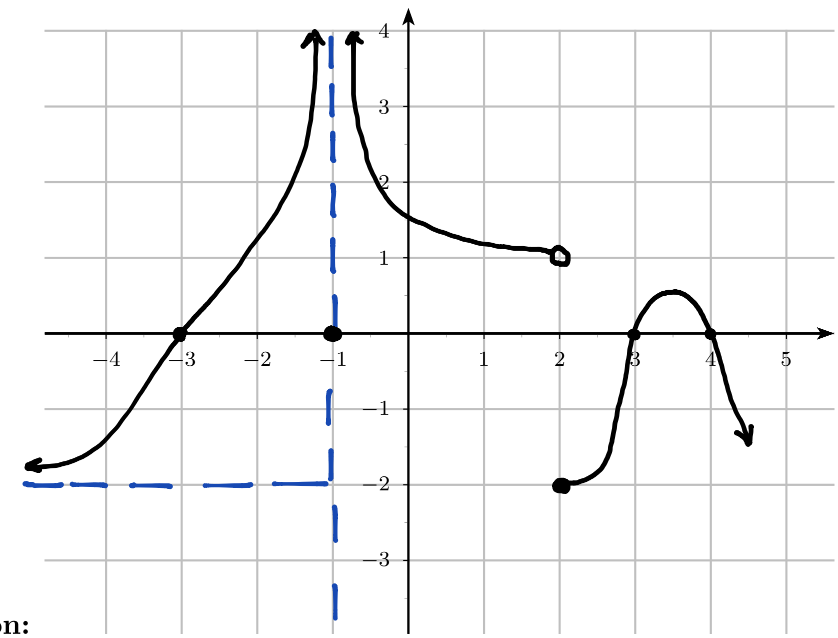
$$\lim_{x\to2^-} f(x) = 2 \qquad \lim_{x\to2^+} f(x) = -2$$

$$f(x) \text{ is continuous from the right at } x = 2$$

$$f(x) \text{ has roots at } x = -1,\ -3,\ 3,\ \text{ and } 4 \text{ only}$$

$$\lim_{x\to+\infty} f(x) = -\infty$$

Solution:



3. Determine $\lim_{x\to\infty} \dfrac{\sqrt{1+4x^2}}{4+x}$

Solution:

$$\lim_{x\to\infty} \frac{\sqrt{1+4x^2}}{4+x} = \lim_{x\to\infty} \frac{\sqrt{(1/x^2)+4}}{(4/x)+1} = \frac{\sqrt{0+4}}{0+1} = 2$$

4. Determine the equation of the tangent line to $y = \dfrac{1}{\sqrt{x}}$ at $(1, 1)$

Solution:

$$\begin{aligned} slope &= \lim_{x\to1} \frac{1/\sqrt{x}-1}{x-1} = \lim_{x\to-1} \frac{-(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)(\sqrt{x}+1)} \\ &= \lim_{x\to1} \frac{-1}{\sqrt{x}(\sqrt{x}+1)} = -\frac{1}{2} \end{aligned}$$

So, the equation of the tangent line is $y - 1 = -\frac{1}{2}(x-1)$.