

Test 2 Review

11. Let $t = \sin x$. Then as $x \rightarrow \pi^-$, $\sin x \rightarrow 0^+$, so $t \rightarrow 0^+$. Thus, $\lim_{x \rightarrow \pi^-} \ln(\sin x) = \lim_{t \rightarrow 0^+} \ln t = -\infty$.

$$12. \lim_{x \rightarrow -\infty} \frac{1-2x^2-x^4}{5+x-3x^4} = \lim_{x \rightarrow -\infty} \frac{(1-2x^2-x^4)/x^4}{(5+x-3x^4)/x^4} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{x^4}-2/x^2-1}{\frac{5}{x^4}+1/x^3-3} = \frac{0-0-1}{0+0-3} = \frac{-1}{-3} = \frac{1}{3}$$

$$\begin{aligned}
 13. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x) &= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2 + 4x + 1} - x}{1} \cdot \frac{\sqrt{x^2 + 4x + 1} + x}{\sqrt{x^2 + 4x + 1} + x} \right] \\
 &= \lim_{x \rightarrow \infty} \frac{(x^2 + 4x + 1) - x^2}{\sqrt{x^2 + 4x + 1} + x} \\
 &= \lim_{x \rightarrow \infty} \frac{(4x + 1)/x}{(\sqrt{x^2 + 4x + 1} + x)/x} \quad [\text{divide by } x = \sqrt{x^2} \text{ for } x > 0] \\
 &= \lim_{x \rightarrow \infty} \frac{4 + 1/x}{\sqrt{1 + 4/x + 1/x^2} + 1} = \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{4}{2} = 2
 \end{aligned}$$

14. Let $t = x - x^2 = x(1-x)$. Then as $x \rightarrow \infty$, $t \rightarrow -\infty$, and $\lim_{x \rightarrow \infty} e^{x-x^2} = \lim_{t \rightarrow -\infty} e^t = 0$.

15. Let $t = 1/x$. Then as $x \rightarrow 0^+$, $t \rightarrow \infty$, and $\lim_{x \rightarrow 0^+} \tan^{-1}(1/x) = \lim_{t \rightarrow \infty} \tan^{-1} t = \frac{\pi}{2}$.

23. $f(x) = 2x^3 + x^2 + 2$ is a polynomial, so it is continuous on $[-2, -1]$ and $f(-2) = -10 < 0 < 1 = f(-1)$. So by the Intermediate Value Theorem there is a number c in $(-2, -1)$ such that $f(c) = 0$, that is, the equation $2x^3 + x^2 + 2 = 0$ has a root in $(-2, -1)$.

24. $f(x) = e^{-x^2} - x$ is continuous on \mathbb{R} so it is continuous on $[0, 1]$. $f(0) = 1 > 0 > 1/e - 1 = f(1)$. So by the Intermediate Value Theorem, there is a number c in $(0, 1)$ such that $f(c) = 0$. Thus, $e^{-x^2} - x = 0$, or $e^{-x^2} = x$, has a root in $(0, 1)$.

25. (a) $s = s(t) = 1 + 2t + t^2/4$. The average velocity over the time interval $[1, 1+h]$ is

$$v_{\text{ave}} = \frac{s(1+h) - s(1)}{(1+h) - 1} = \frac{1 + 2(1+h) + (1+h)^2/4 - 13/4}{h} = \frac{10h + h^2}{4h} = \frac{10 + h}{4}.$$

So for the following intervals the average velocities are:

$$(i) [1, 3]: h = 2, v_{\text{ave}} = (10 + 2)/4 = 3 \text{ m/s} \quad (ii) [1, 2]: h = 1, v_{\text{ave}} = (10 + 1)/4 = 2.75 \text{ m/s}$$

$$(iii) [1, 1.5]: h = 0.5, v_{ave} = (10 + 0.5)/4 = 2.625 \text{ m/s} \quad (iv) [1, 1.1]: h = 0.1, v_{ave} = (10 + 0.1)/4 = 2.525 \text{ m/s}$$

(b) When $t = 1$, the instantaneous velocity is $\lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \rightarrow 0} \frac{10 + h}{4} = \frac{10}{4} = 2.5$ m/s.

27. Estimating the slopes of the tangent lines at $x = 2, 3$, and 5 , we obtain approximate values $0.4, 2$, and 0.1 . Since the graph is concave downward at $x = 5$, $f''(5)$ is negative. Arranging the numbers in increasing order, we have:
 $f''(5) < 0 < f'(5) < f'(2) < 1 < f'(3)$.

28. (a) $f'(2) = \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = \lim_{x \rightarrow 2} \frac{x^3 - 2x - 4}{x - 2}$
 $= \lim_{x \rightarrow 2} \frac{(x - 2)(x^2 + 2x + 2)}{x - 2} = \lim_{x \rightarrow 2} (x^2 + 2x + 2) = 10$

(b) $y - 4 = 10(x - 2)$ or $y = 10x - 16$

(c)

