Name:

Math 560

Limits using the Limit Laws

§2.3

Determine the following limits or indicate that the limit does not exist.

1. $\displaystyle\lim_{x\to -1} \frac{x^3+1}{x+1}$

Solution: $\displaystyle\lim_{x\to -1} \frac{x^3+1}{x+1} = 3$

2. $\displaystyle\lim_{x\to 2} \frac{2-x}{x^2-4}$

Solution: $\displaystyle\lim_{x\to 2} \frac{2-x}{x^2-4} = -\frac{1}{4}$

3. $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4}$

Solution: $\displaystyle\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} = \frac{1}{6}$

4. $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x}$

Solution: $\displaystyle\lim_{x\to \frac{\pi}{2}} \frac{\cos x}{\cot x} = 1$

5. $\displaystyle\lim_{x\to 2} \frac{|x-2|}{x-2}$

Solution: DNE. The limit from the left is not the same as the limit from the right.

6. $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$

Solution: $\displaystyle\lim_{x\to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x} = \frac{1}{2\sqrt{5}}$

7. Using the Limit Laws (i.e. algebraically), determine the limits of #1 and #3.

Solution:

$$\begin{aligned}
\lim_{x\to -1} \frac{x^3+1}{x+1} &= \lim_{x\to -1} \frac{(x+1)(x^2-x+1)}{x+1} \\
&= \lim_{x\to -1} (x^2-x+1) \\
&= (-1)^2-(-1)+1 = 3
\end{aligned}$$

$$\begin{aligned}
\lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} &= \lim_{x\to 4} \frac{\sqrt{x+5}-3}{x-4} \cdot \frac{\sqrt{x+5}+3}{\sqrt{x+5}+3} \\
&= \lim_{x\to 4} \frac{x+5-9}{(x-4)(\sqrt{x+5}+3)} \\
&= \lim_{x\to 4} \frac{x-4}{(x-4)(\sqrt{x+5}+3)} \\
&= \lim_{x\to 4} \frac{1}{\sqrt{x+5}+3} \\
&= \frac{1}{3+3} = \frac{1}{6}
\end{aligned}$$

8. Use the Squeeze Theorem to determine $\displaystyle\lim_{x\to 0} x\cos\frac{1}{x}$.

Solution: Because $-x \leq x \cos \frac{1}{x} \leq x$ and $\lim_{x \to 0} -x = \lim_{x \to 0} x = 0$ it follows by the Squeeze Theorem that $\lim_{x \to 0} x \cos \dfrac{1}{x} = 0$

9. Determine the constant k such that the following limit exists.

$$\lim_{x \to 1} \frac{x^2 - kx + 4}{x - 1}$$

Solution: $k = 5$

10. Determine the constant k such that $\lim_{x \to 0} f(x)$ exists, given that

$$f(x) = \begin{cases} ke^x & \text{if} \quad -1 < x < 0 \\ 2 & \text{if} \quad x = 0 \\ \cos x & \text{if} \quad 0 < x < 1 \end{cases}$$

Solution: $k = 1$