Concepts:

1. *Derivatives of various functions*

   (a) Power Rule: $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$ (n real number)

   (b) Exponential functions: $\frac{d}{dx}\left(a^x\right)=a^x\cdot\ln\left(a\right)$. Note that $\frac{d}{dx}\left(e^x\right)=e^x$

   (c) Trigonometric functions:
   - $\frac{d}{dx}\left(\sin x\right)=\cos x$
   - $\frac{d}{dx}\left(\cos x\right)=-\sin x$
   - $\frac{d}{dx}\left(\tan x\right)=\sec^2 x$
   - $\frac{d}{dx}\left(\cot x\right)=-\csc^2 x$
   - $\frac{d}{dx}\left(\sec x\right)=\sec x\cdot\tan x$
   - $\frac{d}{dx}\left(\csc x\right)=-\csc x\cdot\cot x$

   (d) Derivatives of inverse trigonometric functions
   - $\frac{d}{dx}\sin^{-1}x=\frac{1}{\sqrt{1-x^2}}$
   - $\frac{d}{dx}\cos^{-1}x=-\frac{1}{\sqrt{1-x^2}}$
   - $\frac{d}{dx}\tan^{-1}x=\frac{1}{1+x^2}$

2. *Rules of differentiation (f and g both differentiable)*

   (a) Constant Multiple Rule: $\frac{d}{dx}\left(cf\left(x\right)\right)=c\frac{d}{dx}f\left(x\right)$

   (b) Sum Rule: $\frac{d}{dx}\left(f\left(x\right)+g\left(x\right)\right)=\frac{d}{dx}f\left(x\right)+\frac{d}{dx}g\left(x\right)$

   (c) Difference Rule: $\frac{d}{dx}\left(f\left(x\right)-g\left(x\right)\right)=\frac{d}{dx}f\left(x\right)-\frac{d}{dx}g\left(x\right)$

   (d) Product Rule: $\frac{d}{dx}\left(f\left(x\right)\cdot g\left(x\right)\right)=f\left(x\right)\cdot\frac{d}{dx}g\left(x\right)+g\left(x\right)\cdot\frac{d}{dx}f\left(x\right)$

(e) Quotient Rule: $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\cdot\frac{d}{dx}f(x)-f(x)\cdot\frac{d}{dx}g(x)}{\left(g(x)\right)^2}$

(f) Chain Rule:

i. $(f\circ g)'(x)=f'\left(g(x)\right)\cdot g'(x)$

ii. If $y=f(u)$ and $u=g(x)$, $\frac{dy}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$

(g) Implicit differentiation: To differentiate an expression with x and y when y may not be a function of x, differentiate both sides with respect to x and solve for $y'=\frac{dy}{dx}$ (and use the chain rule when deriving y)

- Example: $x^3+y^3=6xy$. Differentiate with respect to x to get

  $3x^2+3y^2\frac{dy}{dx}=6y+6x\frac{dy}{dx}\Leftrightarrow\frac{dy}{dx}=\frac{2y-x^2}{y^2-2x}$

3. *Some applications of derivatives*

(a) Particle moves forward or backwards according to function $s(t)$ (t is time)

(b) Velocity of the particle is defined by $\frac{ds}{dt}$

- Particle moves forward if velocity is positive
- Particle moves backwards if velocity is negative

(c) Acceleration of the particle is defined by $\frac{dv}{dt}=\frac{d^2s}{dt^2}$

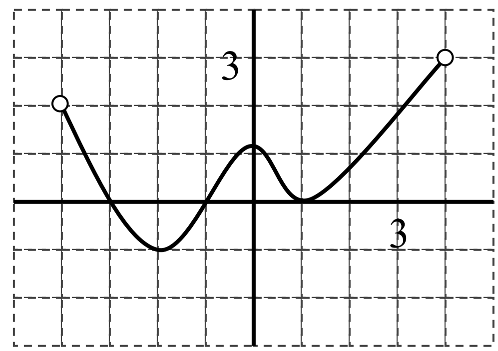
- Particle is speeding up if velocity and acceleration have the same sign
- Particle is slowing down if velocity and acceleration have opposite signs

Review Problems:

p. 255 #3, 4, 5, 30, 31, 37, 40, 47, 48
p.255 #10-13, 16

Additional review problems:

1. Given the graph of the derivative of a function *f* shown on the right
   a) Determine the interval(s) on which *f* is increasing or decreasing
   b) Determine the interval(s) on which *f* is concave up or concave down
   c) For what value(s) of x is the tangent line to *f* horizontal?
   d) If $f(-2)=4$, write an equation of the line tangent to *f* at $x=-2$
   e) Given that $f(0)=-2$, sketch a possible graph of $f(x)$



2. Use the limit definition of a derivative to determine $f'(x)$ if $f(x)=\sqrt{3x-1}$

3. A particle moves along the x-axis so that its position at time t is given by $s(t)=2t^3-9t^2+12t-4$
   a) Determine the velocity of the particle $v(t)$ at time *t*.
   b) Determine the acceleration of the particle $a(t)$ at time *t*.
   c) When is the particle moving to the left?
   d) Find the total distance traveled by the particle between $t=0$ and $t=4$
   e) When is the particle speeding up? When is the particle slowing down?

4. Consider the function $f(x)=x^2\cdot\sin x$ on the interval $[-2\pi,2\pi]$
   a) Find $f'(x)$ and $f''(x)$
   b) Write equations of the tangent lines at $x=\pi$ and $x=-\pi$
   c) On what interval(s) is $f(x)$ increasing or decreasing? Concave up or concave down?

ANSWERS:

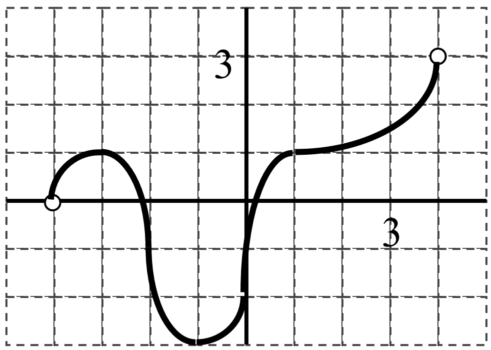
1. (a) Increasing on $(-4,-3)\cup(-1,4)$, decreasing on $(-3,-1)$

   (b) Concave up on $(-2,0)\cup(1,4)$, concave down on $(-4,-2)\cup(0,1)$

   (c) $x=-3,-1,+1$

   (d) $y-4=-(x+2)$

   (e)



2.

$$\lim_{h\to 0}\frac{\sqrt{3(x+h)-1}-\sqrt{3x-1}}{h}\cdot\frac{\sqrt{3(x+h)-1}+\sqrt{3x-1}}{\sqrt{3(x+h)-1}+\sqrt{3x-1}}=$$

$$\lim_{h\to 0}\frac{3h}{h\left(\sqrt{3(x+h)-1}+\sqrt{3x-1}\right)}=\frac{3}{2\sqrt{3x-1}}$$

3. (a) $v(t)=6t^2-18t+12=6(t-2)(t-1)$

   (b) $a(t)=12t-18=6(2t-3)$

   (c) $v(t)<0\Rightarrow(1,2)$

   (d) $|s(1)-s(0)|+|s(2)-s(1)|+|s(4)-s(2)|=5+1+28=34$

   (e) Speeding up when velocity and acceleration have the same sign:
   $\left(1,\frac{3}{2}\right)\cup(2,+\infty)$

4. (a) $f'(x)=x^2\cos x+2x\sin x$

   $f''(x)=4x\cos x-x^2\sin x+2\sin x$

   (b) $f'(\pi)=-\pi^2, y-0=-\pi^2(x-\pi)$

   $f'(-\pi)=-\pi^2, y-0=-\pi^2(x+\pi)$

   (c)[Calc] Concave up: $(-3.994,-1.520)\cup(0,1.520)\cup(3.994,2\pi)$

   Concave down: $(-2\pi,-3.994)\cup(-1.520,0)\cup(1.520,3.994)$