Concepts:

1. *Definition of a derivative*

   (a) The derivative of a function *f* at *a* is $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$.

   (b) $f'(a)$ is also the instantaneous rate of change of $y=f(x)$ with respect to x when $x=a$

   (c) The tangent line to $y=f(x)$ at $(a,f(a))$ is the line through $(a,f(a))$ with slope of $f'(a)$

   (d) The function $nDeriv(Y_1,X,a)$ on your TI-84 lets you calculate numerically the derivative of $Y_1=f(x)$ at a.

2. *The derivative as a function*

   (a) $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

   (b) Notations: $f'(x)=y'=\frac{dy}{dx}=\frac{df}{dx}=\frac{d}{dx}f(x)$

   (c) A function *f* is differentiable at *a* if $f'(a)$ exists. If it is differentiable, then it is continuous (the converse is not true)

   (d) Higher derivatives:

      i. Second derivative: $f''(x)=y''=\frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$

      ii. Third derivative: $f'''(x)=y'''=\frac{d}{dx}\left(\frac{d}{dx}\left(\frac{dy}{dx}\right)\right)=\frac{d^3y}{dx^3}$

      iii. Others: $f^{(n)}(x)=y^{(n)}=\frac{d^ny}{dx^n}$

3. *Understanding the behavior of $f(x)$ from its derivatives*

   (a)

$f(x)$	$f'(x)$	$f''(x)$
Increasing	>0	
Decreasing	<0	
Local Min/Max	when $f'(x)$ changes sign	
Concave Up	Increasing	>0
Concave Down	Decreasing	<0
Point of inflection	Local Min/Max	when $f''(x)$ changes sign

   (b) An antiderivative of *f* is a function *F* such that $F'=f$

Review problems:

p. 176 #26, 28, 31
p. 176 #32, 33, 34
p. 176 #35, 37, 38
p. 176 #41, 42, 46

Additional Problems:

1. Determine the interval(s) on which function $f(x)=2^{x^2-x}$ is increasing.

2. Determine the equations of the two lines tangent to the graph of $f(x)=\frac{1}{x}$ that also go through the point $P(-1,3)$

3. Let $f(x)=x^3+ax^2+bx+c$ with $a\geq 0, b\geq 0$
   a) Over what intervals is *f* concave up, and concave down?
   b) Show that *f* must have at least one inflection point
   c) Given that $(0,-2)$ is the inflection point of *f*, determine *a* and *c*.

Answers for additional problems:

Question 1:

$f'(x) = 2^{x^2-x} \cdot \ln 2 \cdot (2x-1)$

Only the last term controls the sign of the derivative, so the function is increasing when $x > \frac{1}{2}$

Question 2:

$f'(x) = -\frac{1}{x^2}$

Given a point $\left(a, \frac{1}{a}\right)$, the line tangent to $f(x)$ has equation $y - \frac{1}{a} = -\frac{1}{a^2}(x-a)$

This line goes through $(-1,3)$ so $3 - \frac{1}{a} = -\frac{1}{a^2}(-1-a)$

Simplify expression to $3a^2 - 2a - 1 = 0 \Leftrightarrow a = 1, -\frac{1}{3}$

$$a = 1 \to m = \frac{3-1}{-1-1} \to y - 1 = -1(x-1) \Leftrightarrow y = -x + 2$$

$$a = -\frac{1}{3} \to m = \frac{3-(-3)}{-1-(-\frac{1}{3})} = -9 \to y - 3 = -9(x+1) \Leftrightarrow y = -9x - 6$$

Question 3:

Part (a) $f'(x) = 3x^2 + 2ax + b$
$f''(x) = 6x + 2a$

Concave up when $x > -\frac{a}{3}$, concave down otherwise

Part (b) Point of inflection when second derivative changes sign: $x = -\frac{a}{3}$

Part (c) $-\frac{a}{3} = 0 \Rightarrow a = 0, f(0) = c \Rightarrow c = -2$