Concepts:

1. *Limit of a function*
   (a) Definition and notation: $\lim_{x\to a} f(x) = L$
   (b) One-sided limits: $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$
   (c) $\lim_{x\to a} f(x) = L$ if and only if $\lim_{x\to a^-} f(x) = L$ and $\lim_{x\to a^+} f(x) = L$
   (d) Note that $f(a)$ does not have to exist for $\lim_{x\to a} f(x)$ to exist. And even if it exists, $\lim_{x\to a} f(x)$ is not always $f(a)$

2. *Limit laws*
   Given that *c* is a constant, and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist…
   (a) Sum: $\lim_{x\to a}\left(f(x)+g(x)\right) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
   (b) Difference: $\lim_{x\to a}\left(f(x)-g(x)\right) = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
   (c) Constant Multiple: $\lim_{x\to a}\left(cf(x)\right) = c\lim_{x\to a} f(x)$
   (d) Product: $\lim_{x\to a}\left(f(x)\cdot g(x)\right) = \lim_{x\to a} f(x)\cdot \lim_{x\to a} g(x)$
   (e) Quotient: $\lim_{x\to a}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$
   (f) Power: $\lim_{x\to a}\left(f(x)\right)^n = \left(\lim_{x\to a} f(x)\right)^n$ when *n* is a positive integer
   (g) Root: $\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ when *n* is a positive integer
   (h) Squeeze Theorem: if $f(x) \le g(x) \le h(x)$ when *x* is near *a* (not necessarily at *a*), and $\lim_{x\to a} f(x) = L$ and $\lim_{x\to a} h(x) = L$, then $\lim_{x\to a} g(x) = L$

3. *Continuity*
   (a) Definition: *f* continuous at *a* if $\lim_{x\to a} f(x) = f(a)$
   (b) Continuity from left / right: $\lim_{x\to a^-} f(x) = f(a)$, $\lim_{x\to a^+} f(x) = f(a)$
   (c) Examples:
      i. Polynomials are continuous over $\mathbb{R}$
      ii. Rational functions are continuous over their domain
   (d) Continuity when composing functions: if *g* is continuous at *a* and *f* is continuous at $g(a)$, then $(f\circ g)(x) = f\left(g(x)\right)$ is continuous at *a*.

(e) Intermediate Value Theorem: Given that *f* is continuous on interval $[a,b]$, and *N* any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number *c* in (a,b) such that $f(c) = N$ (Paraphrase: if f is continuous on $[a,b]$, then f takes on every value between $f(a)$ and $f(b)$)

4. *Limits involving infinity*
   (a) Notation:
      i. Undefined limit (discontinuity): $\lim_{x \to a} f(x) = \pm\infty$
      ii. Limit of a function as it goes toward infinity (horizontal asymptote): $\lim_{x \to \pm\infty} f(x) = a$
   (b) Discontinuities:
      i. Removable: $f(x) = \begin{cases} 1 - x^2, x \neq 0 \\ 0, x = 0 \end{cases}$
      ii. Jump: $f(x) = \dfrac{e^{1/x}}{e^{1/x} + 1}$
      iii. Infinite: $f(x) = x + \dfrac{1}{x}$
      iv. Oscillation: $f(x) = \sin\left(\dfrac{1}{x}\right)$

5. *Tangents, Velocities and Rates of Change*
   (a) Average rate of change: $\dfrac{f(b) - f(a)}{b - a}$ or $\dfrac{f(x+h) - f(x)}{h}$
   (b) Instantaneous rate of change: $\lim_{h \to 0} \dfrac{f(a+h) - f(a)}{h}$. This limit also becomes the derivative of the function f at $x = a$

6. *Special limits (that you should know)*
   (a) $\lim_{x \to 0} \dfrac{\sin x}{x} = 1$

Review Problems (Stewart):
p. 176 #1, 2, 4, 5, 6, 9, 10, 11, 12, 13, 14, 21, 22, 23, 25

Additional Review Problems:

1. Evaluate the following limits without a calculator:

   (a) $\lim_{x\to -2} \frac{x+2}{x^2-4}$

   (b) $\lim_{x\to 3} \left(\frac{\frac{1}{5x} - \frac{1}{15}}{x-3} \right)$

   (c) $\lim_{t\to 0} \frac{2t}{t+5\tan t}$

   (d) $\lim_{x\to \infty} \frac{x^3-4x+1}{2x^3-5}$

   (e) $\lim_{x\to -\infty} \frac{3x}{\sqrt{3x^2-4}}$

   (f) $\lim_{x\to 0} \frac{\cos x - 1}{\sin^2 x}$

2. Use algebra and limit laws to evaluate the following limit in exact form:
   $\lim_{x\to 2} \frac{\sqrt{x}-\sqrt{2}}{x-2}$

3. Find k such that the function $f(x) = \begin{cases} \cos(kx), x \le 0 \\ \ln(x+k), 0 < x < 3 \end{cases}$ is continuous at $x = 0$

4. Let $f(x) = \begin{cases} mx+b & \text{if } x \le 1 \\ \sqrt{x+3} & \text{if } x > 1 \end{cases}$. Find m and b such that f is differentiable at all real numbers.

Answers:

1. (a) $-\frac{1}{4}$

   (b) $-\frac{1}{45}$

   (c) $\frac{1}{3}$

   (d) $\frac{1}{2}$

   (e) $-\sqrt{3}$

   (f) $-\frac{1}{2}$
2. $\frac{\sqrt{2}}{4}$
3. $k = e$
4. $m = \frac{1}{4}, b = \frac{7}{4}$