Spring 2013 Name:

Math 560

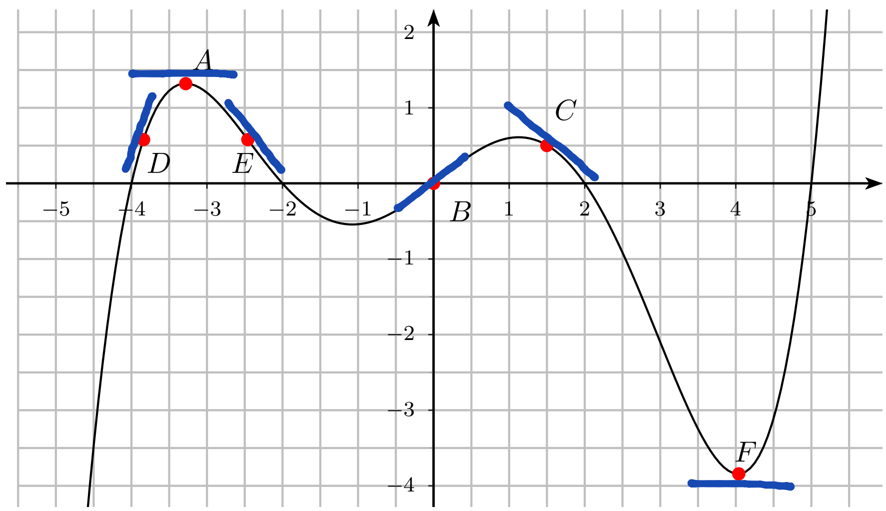
Derivatives

§2.7

1. Using the graph below.
   (a) Draw a small tangent line at each point $A - F$.
   (b) Arrange the following numbers in increasing order.

$$f'(A) \quad f'(B) \quad f'(C) \quad f'(D) \quad f'(E) \quad f'(F)$$

Solution: $f'(E) < f'(C) < f'(A) = f'(F) < f'(B) < f'(D)$



2. What is the value of the slope of the tangent line at each local maximum and minimum point in the above graph?

Solution: At each local max/min the slope of the curve is zero.

3. The temperature, $T(t)$ in degrees, in Boston t hours after midnight on a day last week is given in the table below.

t	0	2	4	6	8	10	12	14
T	53	53	50	49	52	61	68	71

   (a) Estimate the average rate of the temperature change on the following intervals.
      i. $[6, 8]$

Solution: $\dfrac{52-49}{8-6} = \dfrac{3}{2}$

      ii. $[8, 10]$

Solution: $\dfrac{61-52}{10-8} = \dfrac{9}{2}$

(b) Estimate $T'(8)$.

Solution: $\dfrac{\frac{3}{2}+\frac{9}{2}}{2} = 3$

(c) What are the units of part (b)?

Solution: degrees per hour

(d) What is the meaning of the number $T'(8)$?

Solution: $T'(8)$ is the rate (how fast or slow) the temperature is rising per hour at 8a.m.

4. Determine the equation of the tangent line to the graph of $f(x) = 3 - 4x + 2x^2$ at

(a) $x = a$

Solution: You have two ways to do this. I'll use $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

$$\begin{aligned}
\lim_{h\to 0} \frac{f(a+h)-f(a)}{h} &= \lim_{h\to 0} \frac{3-4(a+h)+2(a+h)^2-(3-4a+2a^2)}{h} \\
&= \lim_{h\to 0} \frac{3-4a-4h+2a^2+4ah+2h^2-3+4a-2a^2}{h} \\
&= \lim_{h\to 0} \frac{2h^2+4ah-4h}{h} = \lim_{h\to 0} \frac{h(2h+4a-4)}{h} \\
&= \lim_{h\to 0} 2h+4a-4 = 4a-4
\end{aligned}$$

(b) $x = 0$

Solution: Substitute into the answer for part (a) to get $4(0) - 4 = -4$.

(c) $x = 1$

Solution: Substitute into the answer for part (a) to get $4(1) - 4 = 0$.

(d) $x = 2$

Solution: Substitute into the answer for part (a) to get $4(2) - 4 = 4$.