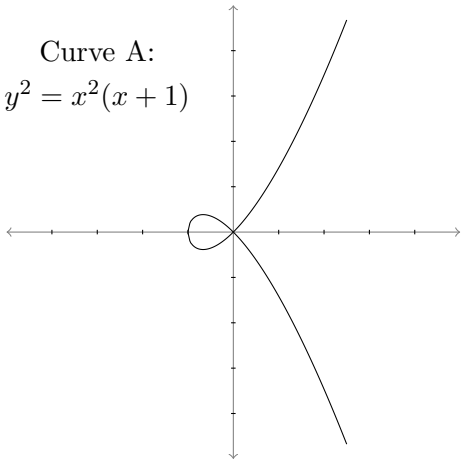
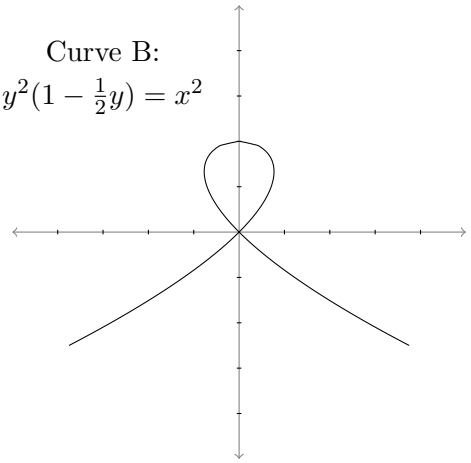
Math 560

Implicit Differentiation

§3.6





1. (a) Use implicit differentiation to find all the points in Curve A with a horizontal tangent line. (Looking at the graph, how many such points should there be?)

Solution: Using implicit differentiation, we get $\frac{dy}{dx} = \frac{x(3x+2)}{2y}$, so the points where $dy/dx = 0$ are those with x-coordinate $-\frac{2}{3}$. (We can't set $x = 0$, because then the equation for Curve A says that y is also 0. Then dy/dx has a zero in the denominator.) Solving for y, we get the two points $\left(-\frac{2}{3}, \frac{4}{27}\right)$ and $\left(-\frac{2}{3}, -\frac{4}{27}\right)$. ($\frac{4}{27} \approx .148148....$)

(b) Use implicit differentiation to find all the points in Curve B with a horizontal tangent line. (Looking at the graph, how many such points should there be?)

Solution: Using implicit differentiation, we get

$$\frac{dy}{dx} = \frac{2x}{2y - \frac{3}{2}y^2},$$

so the points where $dy/dx = 0$ are those with $x = 0$ and y nonzero. If $x = 0$ on the curve, then either $y = 0$ or $y = 2$, so we have only the point $(0, 2)$.

(c) Try to find $\frac{dy}{dx}$ at the point $(0, 0)$ on both graphs. What goes wrong?

Solution: It's not possible to plug in $x = 0$ and $y = 0$ to either of the expressions for $\frac{dy}{dx}$. The derivative function tells us nothing about $(0, 0)$.