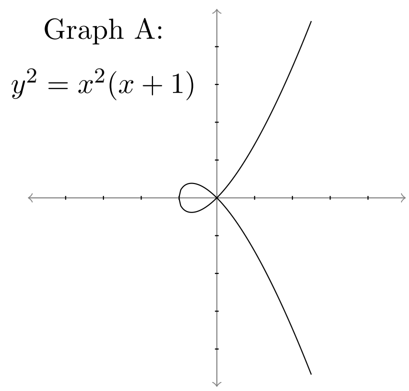
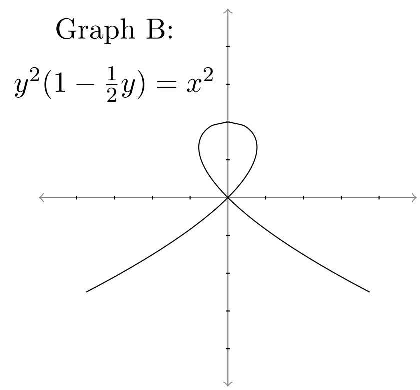
2. In this problem you'll look at the curves from Page 1 in a different way.

Suppose a cat is chasing a ball around on the floor, and its position is described by the parametric equations

$$(x(t), y(t)) = (t^2 - 1, t - t^3).$$





(a) The cat is following one of the paths from the previous page (reprinted above). Which path does the cat follow? Circle this curve. How do you know it's the right one?

> **Solution:** The expressions for $x(t)$ and $y(t)$ satisfy the equation corresponding to Curve A. That is, $y(t)^2 = x(t)^2(x(t)+1)$.

(b) Draw an arrow on the circled graph above, to indicate in which direction the cat is running.

> **Solution:** As long as t is bigger than 1, the expression $y(t) = t - t^3$ is negative, so the cat eventually stays in the lower half of the plane. The cat enters the picture in the upper right, runs around a loop near the origin, and runs away to the lower right.

(c) At which time(s) t does the cat run through the point $(0,0)$?

> **Solution:** If $x(t) = 0$, then $t = \pm 1$, and if $y = 0$ then $t = \pm 1$ or $t = 0$. So the only times when x and y are both zero are $t = \pm 1$.

(d) Remember that it wasn't possible to find $\frac{dy}{dx}$ at $(0,0)$ using the method on Page 1. But now that the graph has been parametrized, you can do it. What are the tangent line(s) to the parametrized curve $(x(t), y(t))$ at $(0,0)$?

Solution: We'll use the chain rule, which says that

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Now

$$\frac{dy}{dt} = 1 - 3t^2, \qquad \frac{dx}{dt} = 2t.$$

At $t = 1$ we have $\frac{dy}{dt} = -2$ and $\frac{dx}{dt} = 2$; at $t = -1$ we have $\frac{dy}{dt} = -2$ and $\frac{dx}{dt} = -2$. So the slopes are -1 and 1, respectively, and the equations of the tangent lines are

$$y = -x \qquad \text{and} \qquad y = x.$$