

Final Review 3

3. $y = \sqrt{x} + \frac{1}{\sqrt[3]{x^4}} = x^{1/2} + x^{-4/3} \Rightarrow y' = \frac{1}{2}x^{-1/2} - \frac{4}{3}x^{-7/3} = \frac{1}{2\sqrt{x}} - \frac{4}{3\sqrt[3]{x^7}}$

4. $y = \frac{3x-2}{\sqrt{2x+1}} \Rightarrow$

$$y' = \frac{\sqrt{2x+1}(3) - (3x-2)\frac{1}{2}(2x+1)^{-1/2}(2)}{(\sqrt{2x+1})^2} \cdot \frac{(2x+1)^{1/2}}{(2x+1)^{1/2}} = \frac{3(2x+1) - (3x-2)}{(2x+1)^{3/2}} = \frac{3x+5}{(2x+1)^{3/2}}$$

5. $y = 2x\sqrt{x^2+1} \Rightarrow$

$$y' = 2x \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x) + \sqrt{x^2+1}(2) = \frac{2x^2}{\sqrt{x^2+1}} + 2\sqrt{x^2+1} = \frac{2x^2 + 2(x^2+1)}{\sqrt{x^2+1}} = \frac{2(2x^2+1)}{\sqrt{x^2+1}}$$

10. $y = \sin^{-1}(e^x) \Rightarrow y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = e^x / \sqrt{1-e^{2x}}$

11. $y = xe^{-1/x} \Rightarrow y' = xe^{-1/x}(1/x^2) + e^{-1/x} \cdot 1 = e^{-1/x}(1/x + 1)$

12. $y = x^r e^{sx} \Rightarrow y' = x^r(se^{sx}) + e^{sx}(rx^{r-1}) = e^{sx}x^{r-1}(sx+r)$

13. $\frac{d}{dx}(xy^4 + x^2y) = \frac{d}{dx}(x+3y) \Rightarrow x \cdot 4y^3y' + y^4 \cdot 1 + x^2 \cdot y' + y \cdot 2x = 1 + 3y' \Rightarrow$
 $y'(4xy^3 + x^2 - 3) = 1 - y^4 - 2xy \Rightarrow y' = \frac{1 - y^4 - 2xy}{4xy^3 + x^2 - 3}$

16. $\frac{d}{dx}(x^2 \cos y + \sin 2y) = \frac{d}{dx}(xy) \Rightarrow x^2(-\sin y \cdot y') + (\cos y)(2x) + \cos 2y \cdot 2y' = x \cdot y' + y \cdot 1 \Rightarrow$
 $y'(-x^2 \sin y + 2 \cos 2y - x) = y - 2x \cos y \Rightarrow y' = \frac{y - 2x \cos y}{2 \cos 2y - x^2 \sin y - x}$

30. $y = 10^{\tan \pi \theta} \Rightarrow y' = 10^{\tan \pi \theta} \cdot \ln 10 \cdot \sec^2 \pi \theta \cdot \pi = \pi(\ln 10)10^{\tan \pi \theta} \sec^2 \pi \theta$

31. $y = \tan^2(\sin \theta) = [\tan(\sin \theta)]^2 \Rightarrow y' = 2[\tan(\sin \theta)] \cdot \sec^2(\sin \theta) \cdot \cos \theta$

37. $f(x) = 2^x \Rightarrow f'(x) = 2^x \ln 2 \Rightarrow f''(x) = (2^x \ln 2) \ln 2 = 2^x (\ln 2)^2 \Rightarrow$

$$f'''(x) = (2^x \ln 2)(\ln 2)^2 = 2^x (\ln 2)^3 \Rightarrow \dots \Rightarrow f^{(n)}(x) = (2^x \ln 2)(\ln 2)^{n-1} = 2^x (\ln 2)^n$$

40. $y = \frac{x^2-1}{x^2+1} \Rightarrow y' = \frac{(x^2+1)(2x) - (x^2-1)(2x)}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}.$

At $(0, -1)$, $y' = 0$, so an equation of the tangent line is $y + 1 = 0(x - 0)$, or $y = -1$.

47. (a) $h(x) = f(x)g(x) \Rightarrow h'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow$

$$h'(2) = f(2)g'(2) + g(2)f'(2) = (3)(4) + (5)(-2) = 12 - 10 = 2$$

(b) $F(x) = f(g(x)) \Rightarrow F'(x) = f'(g(x))g'(x) \Rightarrow F'(2) = f'(g(2))g'(2) = f'(5)(4) = 11 \cdot 4 = 44$

48. (a) $P(x) = f(x)g(x) \Rightarrow P'(x) = f(x)g'(x) + g(x)f'(x) \Rightarrow$
 $P'(2) = f(2)g'(2) + g(2)f'(2) = (1)\left(\frac{6-0}{3-0}\right) + (4)\left(\frac{0-3}{3-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2$
- (b) $Q(x) = \frac{f(x)}{g(x)} \Rightarrow Q'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \Rightarrow$
 $Q'(2) = \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(4)(-1) - (1)(2)}{4^2} = \frac{-6}{16} = -\frac{3}{8}$
- (c) $C(x) = f(g(x)) \Rightarrow C'(x) = f'(g(x))g'(x) \Rightarrow$
 $C'(2) = f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6$