

1. (a) (i) $\lim_{x \rightarrow 2^+} f(x) = 3$

(ii) $\lim_{x \rightarrow -3^+} f(x) = 0$

(iii) $\lim_{x \rightarrow -3} f(x)$ does not exist since the left and right limits are not equal. (The left limit is -2 .)

(iv) $\lim_{x \rightarrow 4} f(x) = 2$

(v) $\lim_{x \rightarrow 0} f(x) = \infty$

(vi) $\lim_{x \rightarrow 2^-} f(x) = -\infty$

(vii) $\lim_{x \rightarrow \infty} f(x) = 4$

(viii) $\lim_{x \rightarrow -\infty} f(x) = -1$

(b) The equations of the horizontal asymptotes are $y = -1$ and $y = 4$.

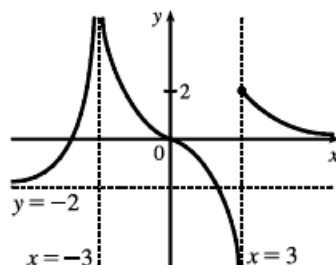
(c) The equations of the vertical asymptotes are $x = 0$ and $x = 2$.

(d) f is discontinuous at $x = -3, 0, 2$, and 4 . The discontinuities are jump, infinite, infinite, and removable, respectively.

2. $\lim_{x \rightarrow -\infty} f(x) = -2$, $\lim_{x \rightarrow \infty} f(x) = 0$, $\lim_{x \rightarrow -3} f(x) = \infty$,

$\lim_{x \rightarrow 3^-} f(x) = -\infty$, $\lim_{x \rightarrow 3^+} f(x) = 2$,

f is continuous from the right at 3



4. Since rational functions are continuous, $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 + 2x - 3} = \frac{3^2 - 9}{3^2 + 2(3) - 3} = \frac{0}{12} = 0$.

5. $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x^2 + 2x - 3} = \lim_{x \rightarrow -3} \frac{(x+3)(x-3)}{(x+3)(x-1)} = \lim_{x \rightarrow -3} \frac{x-3}{x-1} = \frac{-3-3}{-3-1} = \frac{-6}{-4} = \frac{3}{2}$

6. $\lim_{x \rightarrow 1^+} \frac{x^2 - 9}{x^2 + 2x - 3} = -\infty$ since $x^2 + 2x - 3 \rightarrow 0$ as $x \rightarrow 1^+$ and $\frac{x^2 - 9}{x^2 + 2x - 3} < 0$ for $1 < x < 3$.

9. $\lim_{r \rightarrow 9} \frac{\sqrt{r}}{(r-9)^4} = \infty$ since $(r-9)^4 \rightarrow 0$ as $r \rightarrow 9$ and $\frac{\sqrt{r}}{(r-9)^4} > 0$ for $r \neq 9$.

10. $\lim_{v \rightarrow 4^+} \frac{4-v}{|4-v|} = \lim_{v \rightarrow 4^+} \frac{4-v}{-(4-v)} = \lim_{v \rightarrow 4^+} \frac{1}{-1} = -1$

11. Let $t = \sin x$. Then as $x \rightarrow \pi^-$, $\sin x \rightarrow 0^+$, so $t \rightarrow 0^+$. Thus, $\lim_{x \rightarrow \pi^-} \ln(\sin x) = \lim_{t \rightarrow 0^+} \ln t = -\infty$.

12. $\lim_{x \rightarrow -\infty} \frac{1 - 2x^2 - x^4}{5 + x - 3x^4} = \lim_{x \rightarrow -\infty} \frac{(1 - 2x^2 - x^4)/x^4}{(5 + x - 3x^4)/x^4} = \lim_{x \rightarrow -\infty} \frac{1/x^4 - 2/x^2 - 1}{5/x^4 + 1/x^3 - 3} = \frac{0 - 0 - 1}{0 + 0 - 3} = \frac{-1}{-3} = \frac{1}{3}$

$$\begin{aligned}
 13. \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4x + 1} - x) &= \lim_{x \rightarrow \infty} \left[\frac{\sqrt{x^2 + 4x + 1} - x}{1} \cdot \frac{\sqrt{x^2 + 4x + 1} + x}{\sqrt{x^2 + 4x + 1} + x} \right] \\
 &= \lim_{x \rightarrow \infty} \frac{(x^2 + 4x + 1) - x^2}{\sqrt{x^2 + 4x + 1} + x} \\
 &= \lim_{x \rightarrow \infty} \frac{(4x + 1)/x}{(\sqrt{x^2 + 4x + 1} + x)/x} \quad \left[\text{divide by } x = \sqrt{x^2} \text{ for } x > 0 \right] \\
 &= \lim_{x \rightarrow \infty} \frac{4 + 1/x}{\sqrt{1 + 4/x + 1/x^2} + 1} = \frac{4 + 0}{\sqrt{1 + 0 + 0} + 1} = \frac{4}{2} = 2
 \end{aligned}$$

14. Let $t = x - x^2 = x(1 - x)$. Then as $x \rightarrow \infty$, $t \rightarrow -\infty$, and $\lim_{x \rightarrow \infty} e^{x-x^2} = \lim_{t \rightarrow -\infty} e^t = 0$.

21. (a) $f(x) = \sqrt{-x}$ if $x < 0$, $f(x) = 3 - x$ if $0 \leq x < 3$, $f(x) = (x - 3)^2$ if $x > 3$.

(i) $\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (3 - x) = 3$

(ii) $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} \sqrt{-x} = 0$

(iii) Because of (i) and (ii), $\lim_{x \rightarrow 0} f(x)$ does not exist.

(iv) $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (3 - x) = 0$

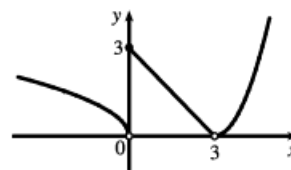
(v) $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (x - 3)^2 = 0$

(vi) Because of (iv) and (v), $\lim_{x \rightarrow 3} f(x) = 0$.

(b) f is discontinuous at 0 since $\lim_{x \rightarrow 0} f(x)$ does not exist.

(c)

f is discontinuous at 3 since $f(3)$ does not exist.



22. (a) $x^2 - 9$ is continuous on \mathbb{R} since it is a polynomial and \sqrt{x} is continuous on $[0, \infty)$, so the composition $\sqrt{x^2 - 9}$ is continuous on $\{x \mid x^2 - 9 \geq 0\} = (-\infty, -3] \cup [3, \infty)$. Note that $x^2 - 2 \neq 0$ on this set and so the quotient function

$g(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}$ is continuous on its domain, $(-\infty, -3] \cup [3, \infty)$.

(b) f is discontinuous at 0 since $\lim_{x \rightarrow 0} f(x)$ does not exist. f is discontinuous at 3 since $f(3)$ does not exist.

23. $f(x) = 2x^3 + x^2 + 2$ is a polynomial, so it is continuous on $[-2, -1]$ and $f(-2) = -10 < 0 < 1 = f(-1)$. So by the Intermediate Value Theorem there is a number c in $(-2, -1)$ such that $f(c) = 0$, that is, the equation $2x^3 + x^2 + 2 = 0$ has a root in $(-2, -1)$.

25. (a) $s = s(t) = 1 + 2t + t^2/4$. The average velocity over the time interval $[1, 1 + h]$ is

$$v_{\text{ave}} = \frac{s(1+h) - s(1)}{(1+h) - 1} = \frac{1 + 2(1+h) + (1+h)^2/4 - 13/4}{h} = \frac{10h + h^2}{4h} = \frac{10 + h}{4}.$$

So for the following intervals the average velocities are:

(i) $[1, 3]$: $h = 2$, $v_{\text{ave}} = (10 + 2)/4 = 3$ m/s

(ii) $[1, 2]$: $h = 1$, $v_{\text{ave}} = (10 + 1)/4 = 2.75$ m/s

(iii) $[1, 1.5]$: $h = 0.5$, $v_{\text{ave}} = (10 + 0.5)/4 = 2.625$ m/s

(iv) $[1, 1.1]$: $h = 0.1$, $v_{\text{ave}} = (10 + 0.1)/4 = 2.525$ m/s

(b) When $t = 1$, the instantaneous velocity is $\lim_{h \rightarrow 0} \frac{s(1+h) - s(1)}{h} = \lim_{h \rightarrow 0} \frac{10 + h}{4} = \frac{10}{4} = 2.5$ m/s.