Math 560

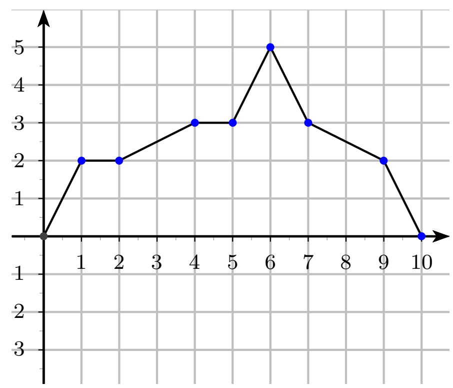
Derivatives as Functions

§§*2.6-2.8*

1. The position of a particle, $s(t)$ where t is time and $0 \le t \le 10$, as it moves back and forth along a horizontal line is shown in the graph below.

   (a) On what interval(s) is the particle moving to the right? Justify your answer.

   Solution: $(0, 1)$ and $(2, 4)$ and $(5, 6)$ because on each of these intervals the particle is getting further away from it starting point.



   (b) On what interval(s) is the particle moving to the left? Justify your answer.

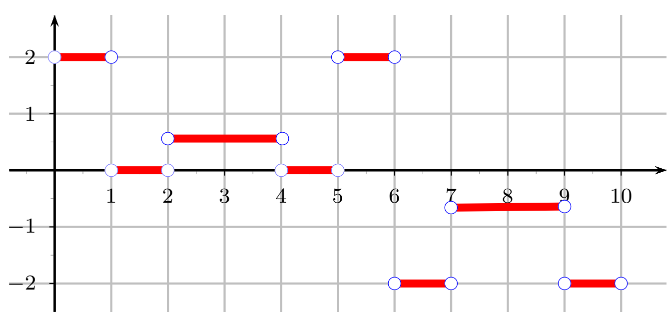
   Solution: $(6, 10)$ because during this time interval the particle is moving towards its starting point.

   (c) On what interval(s) is the particle standing still? Justify your answer.

   Solution: $(1, 2)$ and $(4, 5)$ because its distance isn't changing.

   (d) Sketch a graph of the velocity function.

   Solution:



2. Let $f(x) = 2x^2 - 3x$.

   (a) Determine $f'(x)$ using the definition.

   Solution:

$$\lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - (2x^2 - 3x)}{h} = \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3x - 3h - 2x^2 + 3x}{h}$$
$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 3h}{h} = \lim_{h \to 0} 4x + 2h - 3$$
$$= 4x - 3$$

   (b) Complete the following table.

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$											

Solution:

x	-5	-4	-3	-2	-1	0	1	2	3	4	5
$f'(x)$	-23	-19	-15	-11	-7	-3	1	5	9	13	17

(c) Determine the equation of the tangent line to f at $x = 2$.

Solution: $y - 20 = 5(x - 2)$

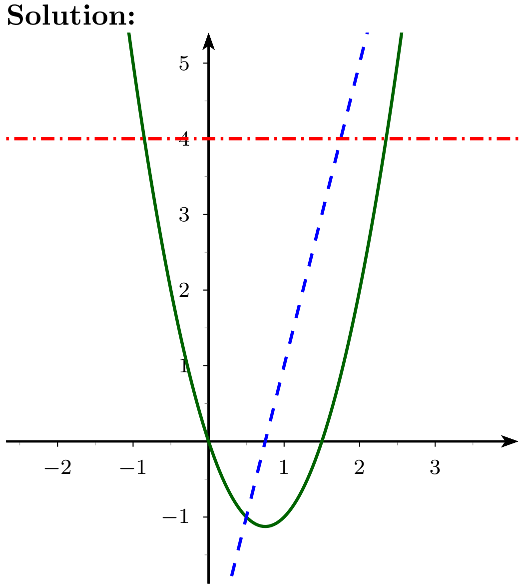
(d) Determine $f''(x)$ using the definition.

Solution:

$$\lim_{h\to 0} \frac{4(x+h) - 3 - 4x + 3}{h} = \lim_{h\to 0} \frac{4h}{h} = \lim_{h\to 0} 4 = 4$$

(e) Sketch the graph of f, $f'$and f'' on the same axes.

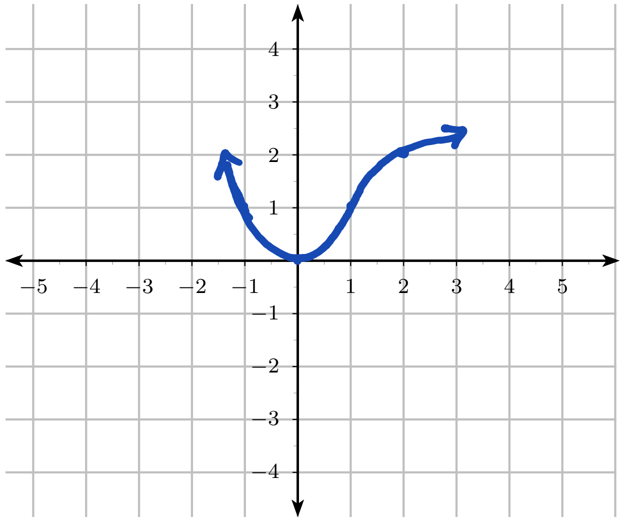
Solution:



3. Sketch the graph of a function f with all the following properties.

$$f(0) = f'(0) = 0$$
$$f'(-1) = -1, \quad f'(1) = 3, \quad f'(2) = 1.$$

Solution:



4. Find approximate values for the derivative of $f(x)$ to complete the table below.

x	0	1	2	3	4	5	6	7	8
$f(x)$	18	13	10	9	9	11	15	21	30
$f'(x)$									

Solution:

x	0	1	2	3	4	5	6	7	8
$f(x)$	18	13	10	9	9	11	15	21	30
$f'(x)$	-5	-4	-2	$\frac{1}{2}$	1	3	5	7.5	9

5. For $f(x) = \sqrt{x+3}$, determine $f'(x)$ using the definition.

Solution:

$$\begin{aligned}
\lim_{h\to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} &= \lim_{h\to 0} \frac{\sqrt{x+h+3} - \sqrt{x+3}}{h} \cdot \frac{\sqrt{x+h+3} + \sqrt{x+3}}{\sqrt{x+h+3} + \sqrt{x+3}} \\
&= \lim_{h\to 0} \frac{(x+h+3) - (x+3)}{h\sqrt{x+h+3} + \sqrt{x+3}} \\
&= \lim_{h\to 0} \frac{1}{\sqrt{x+h+3} + \sqrt{x+3}} \\
&= \frac{1}{2\sqrt{x+3}}
\end{aligned}$$