$$\begin{aligned} \mathbf{2} \ y &= \cos(\tan x) \ \Rightarrow \ y' = -\sin(\tan x) \frac{d}{dx} (\tan x) = -\sin(\tan x)(\sec^2 x) \\ \mathbf{4} \ y &= \frac{3x-2}{\sqrt{2x+1}} \ \Rightarrow \\ y' &= \frac{\sqrt{2x+1}(3) - (3x-2)\frac{1}{2}(2x+1)^{-1/2}(2)}{(\sqrt{2x+1})^2} \cdot \frac{(2x+1)^{1/2}}{(2x+1)^{1/2}} = \frac{3(2x+1) - (3x-2)}{(2x+1)^{3/2}} = \frac{3x+5}{(2x+1)^{3/2}} \\ \mathbf{6} \ y &= \frac{e^x}{1+x^2} \ \Rightarrow \ y' = \frac{(1+x^2)e^x - e^x(2x)}{(1+x^2)^2} = \frac{e^x(x^2 - 2x+1)}{(1+x^2)^2} = \frac{e^x(x-1)^2}{(1+x^2)^2} \\ \mathbf{8} \ y &= e^{-t}(t^2 - 2t + 2) \ \Rightarrow \ y' = e^{-t}(2t-2) + (t^2 - 2t + 2)(-e^{-t}) = e^{-t}(2t-2 - t^2 + 2t - 2) = e^{-t}(-t^2 + 4t - 4) \\ \mathbf{11} \ y &= xe^{-1/x} \ \Rightarrow \ y' = xe^{-1/x}(1/x^2) + e^{-1/x} \cdot 1 = e^{-1/x}(1/x + 1) \\ \mathbf{41} \ y &= (2+x)e^{-x} \ \Rightarrow \ y' &= (2+x)(-e^{-x}) + e^{-x} \cdot 1 = e^{-x}[-(2+x) + 1] = e^{-x}(-x-1). \\ At (0, 2), y' &= 1(-1) = -1, \text{ so an equation of the tangent line is } y - 2 = -1(x-0), \text{ or } y = -x + 2. \\ The slope of the normal line is 1, so an equation of the normal line is $y - 2 = 1(x - 0), \text{ or } y = x + 2. \\ \mathbf{48} \ (a) \ P(x) &= f(x)g(x) \ \Rightarrow \ P'(x) &= f(x)g'(x) + g(x)f'(x) \ \Rightarrow \\ P'(2) &= f(2)g'(2) + g(2)f'(2) = (1)\left(\frac{g-a}{2-0}\right) + (4)\left(\frac{0-2}{2-0}\right) = (1)(2) + (4)(-1) = 2 - 4 = -2 \\ (b) \ Q(x) &= \frac{f(x)}{g(x)} \ \Rightarrow \ Q'(x) &= \frac{g(x)f'(x) - f(x)g'(x)}{|g(x)|^2} \ \Rightarrow \end{aligned}$$$

$$\begin{aligned} Q'(2) &= \frac{g(2)f'(2) - f(2)g'(2)}{[g(2)]^2} = \frac{(4)(-1) - (1)(2)}{4^2} = \frac{-6}{16} = -\frac{3}{8} \\ \text{(c) } C(x) &= f(g(x)) \implies C'(x) = f'(g(x))g'(x) \implies \\ C'(2) &= f'(g(2))g'(2) = f'(4)g'(2) = \left(\frac{6-0}{5-3}\right)(2) = (3)(2) = 6 \end{aligned}$$

53. $f(x) = g(e^x) \Rightarrow f'(x) = g'(e^x)e^x$

- **54.** $f(x) = e^{g(x)} \Rightarrow f'(x) = e^{g(x)}g'(x)$
- 60. (a) The line x 4y = 1 has slope ¹/₄. A tangent to y = e^x has slope ¹/₄ when y' = e^x = ¹/₄ ⇒ x = ln ¹/₄ = -ln 4. Since y = e^x, the y-coordinate is ¹/₄ and the point of tangency is (-ln 4, ¹/₄). Thus, an equation of the tangent line is y ¹/₄ = ¹/₄(x + ln 4) or y = ¹/₄x + ¹/₄(ln 4 + 1).
 - (b) The slope of the tangent at the point (a, e^a) is d/dx e^x | x = a = e^a. Thus, an equation of the tangent line is y e^a = e^a(x a). We substitute x = 0, y = 0 into this equation, since we want the line to pass through the origin: 0 e^a = e^a(0 a) ⇔ -e^a = e^a(-a) ⇔ a = 1. So an equation of the tangent line at the point (a, e^a) = (1, e) is y e = e(x 1) or y = ex.