1. (a) $\dfrac{d}{dx}(xy+2x+3x^2)=\dfrac{d}{dx}(4) \;\Rightarrow\; (x\cdot y'+y\cdot 1)+2+6x=0 \;\Rightarrow\; xy'=-y-2-6x \;\Rightarrow$
$y'=\dfrac{-y-2-6x}{x}$ or $y'=-6-\dfrac{y+2}{x}$.

(b) $xy+2x+3x^2=4 \;\Rightarrow\; xy=4-2x-3x^2 \;\Rightarrow\; y=\dfrac{4-2x-3x^2}{x}=\dfrac{4}{x}-2-3x$, so $y'=-\dfrac{4}{x^2}-3$.

(c) From part (a), $y'=\dfrac{-y-2-6x}{x}=\dfrac{-(4/x-2-3x)-2-6x}{x}=\dfrac{-4/x-3x}{x}=-\dfrac{4}{x^2}-3$.

9. $\dfrac{d}{dx}(4\cos x\sin y)=\dfrac{d}{dx}(1) \;\Rightarrow\; 4\,[\cos x\cdot\cos y\cdot y'+\sin y\cdot(-\sin x)]=0 \;\Rightarrow$
$y'(4\cos x\cos y)=4\sin x\sin y \;\Rightarrow\; y'=\dfrac{4\sin x\sin y}{4\cos x\cos y}=\tan x\tan y$

13. $\dfrac{d}{dx}\{f(x)+x^2[f(x)]^3\}=\dfrac{d}{dx}(10) \;\Rightarrow\; f'(x)+x^2\cdot 3[f(x)]^2\cdot f'(x)+[f(x)]^3\cdot 2x=0$. If $x=1$, we have
$f'(1)+1^2\cdot 3[f(1)]^2\cdot f'(1)+[f(1)]^3\cdot 2(1)=0 \;\Rightarrow\; f'(1)+1\cdot 3\cdot 2^2\cdot f'(1)+2^3\cdot 2=0 \;\Rightarrow$
$f'(1)+12f'(1)=-16 \;\Rightarrow\; 13f'(1)=-16 \;\Rightarrow\; f'(1)=-\frac{16}{13}$.

18. $x^{2/3}+y^{2/3}=4 \;\Rightarrow\; \frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}y'=0 \;\Rightarrow\; \dfrac{1}{\sqrt[3]{x}}+\dfrac{y'}{\sqrt[3]{y}}=0 \;\Rightarrow\; y'=-\dfrac{\sqrt[3]{y}}{\sqrt[3]{x}}$. When $x=-3\sqrt{3}$
and $y=1$, we have $y'=-\dfrac{1}{\left(-3\sqrt{3}\right)^{1/3}}=-\dfrac{\left(-3\sqrt{3}\right)^{2/3}}{-3\sqrt{3}}=\dfrac{3}{3\sqrt{3}}=\dfrac{1}{\sqrt{3}}$, so an equation of the tangent line is
$y-1=\frac{1}{\sqrt{3}}\left(x+3\sqrt{3}\right)$ or $y=\frac{1}{\sqrt{3}}x+4$.

27. (a) $x^4+y^4=16 \;\Rightarrow\; 4x^3+4y^3y'=0 \;\Rightarrow\; y^3y'=-x^3 \;\Rightarrow\; y'=-x^3/y^3$

(b) $y''=-\dfrac{y^3(3x^2)-(x^3)(3y^2y')}{(y^3)^2}=-\dfrac{3x^2y^3-3x^3y^2(-x^3/y^3)}{y^6}\cdot\dfrac{y}{y}=-\dfrac{3x^2y^4+3x^6}{y^7}$

(c) $y''=-\dfrac{3x^2(y^4+x^4)}{y^7}=-\dfrac{3x^2(16)}{y^7}=-48\dfrac{x^2}{y^7}$

52. $x^2+4y^2=36 \;\Rightarrow\; 2x+8yy'=0 \;\Rightarrow\; y'=-\dfrac{x}{4y}$. Let (a,b) be a point on $x^2+4y^2=36$ whose tangent line passes through $(12,3)$. The tangent line is then $y-3=-\dfrac{a}{4b}(x-12)$, so $b-3=-\dfrac{a}{4b}(a-12)$. Multiplying both sides by $4b$ gives $4b^2-12b=-a^2+12a$, so $4b^2+a^2=12(a+b)$. But $4b^2+a^2=36$, so $36=12(a+b) \;\Rightarrow\; a+b=3 \;\Rightarrow$ $b=3-a$. Substituting $3-a$ for b into $a^2+4b^2=36$ gives $a^2+4(3-a)^2=36 \;\Leftrightarrow\; a^2+36-24a+4a^2=36 \;\Leftrightarrow$ $5a^2-24a=0 \;\Leftrightarrow\; a(5a-24)=0$, so $a=0$ or $a=\frac{24}{5}$. If $a=0$, $b=3-0=3$, and if $a=\frac{24}{5}$, $b=3-\frac{24}{5}=-\frac{9}{5}$.
So the two points on the ellipse are $(0,3)$ and $\left(\frac{24}{5},-\frac{9}{5}\right)$. Using $y-3=-\dfrac{a}{4b}(x-12)$ with $(a,b)=(0,3)$ gives us the tangent line $y-3=0$ or $y=3$. With $(a,b)=\left(\frac{24}{5},-\frac{9}{5}\right)$, we have $y-3=-\frac{24/5}{4(-9/5)}(x-12) \;\Leftrightarrow\; y-3=\frac{2}{3}(x-12) \;\Leftrightarrow\; y=\frac{2}{3}x-5$.
A graph of the ellipse and the tangent lines confirms our results.