
3.5 In Class Problems

1. Let $u = g(x) = 4x$ and $y = f(u) = \sin u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\cos u)(4) = 4 \cos 4x$.
2. Let $u = g(x) = 4 + 3x$ and $y = f(u) = \sqrt{u} = u^{1/2}$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = \frac{1}{2}u^{-1/2}(3) = \frac{3}{2\sqrt{u}} = \frac{3}{2\sqrt{4+3x}}$.
3. Let $u = g(x) = 1 - x^2$ and $y = f(u) = u^{10}$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (10u^9)(-2x) = -20x(1 - x^2)^9$.
4. Let $u = g(x) = \sin x$ and $y = f(u) = \tan u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\sec^2 u)(\cos x) = \sec^2(\sin x) \cdot \cos x$,
or equivalently, $[\sec(\sin x)]^2 \cos x$.
5. Let $u = g(x) = \sqrt{x}$ and $y = f(u) = e^u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (e^u)\left(\frac{1}{2}x^{-1/2}\right) = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$.
6. Let $u = g(x) = e^x$ and $y = f(u) = \sin u$. Then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = (\cos u)(e^x) = e^x \cos e^x$.
7. $F(x) = \sqrt[4]{1+2x+x^3} = (1+2x+x^3)^{1/4} \Rightarrow$
$$F'(x) = \frac{1}{4}(1+2x+x^3)^{-3/4} \cdot \frac{d}{dx}(1+2x+x^3) = \frac{1}{4(1+2x+x^3)^{3/4}} \cdot (2+3x^2)$$
$$= \frac{2+3x^2}{4(1+2x+x^3)^{3/4}} = \frac{2+3x^2}{4\sqrt[4]{(1+2x+x^3)^3}}$$
8. $F(x) = (x^2 - x + 1)^3 \Rightarrow F'(x) = 3(x^2 - x + 1)^2(2x - 1)$
9. $g(t) = \frac{1}{(t^4 + 1)^3} = (t^4 + 1)^{-3} \Rightarrow g'(t) = -3(t^4 + 1)^{-4}(4t^3) = -12t^3(t^4 + 1)^{-4} = \frac{-12t^3}{(t^4 + 1)^4}$
10. $f(t) = \sqrt[3]{1+\tan t} = (1+\tan t)^{1/3} \Rightarrow f'(t) = \frac{1}{3}(1+\tan t)^{-2/3} \sec^2 t = \frac{\sec^2 t}{3\sqrt[3]{(1+\tan t)^2}}$
11. $y = \cos(a^3 + x^3) \Rightarrow y' = -\sin(a^3 + x^3) \cdot 3x^2$ [a^3 is just a constant] $= -3x^2 \sin(a^3 + x^3)$
12. $y = a^3 + \cos^3 x \Rightarrow y' = 3(\cos x)^2(-\sin x)$ [a^3 is just a constant] $= -3 \sin x \cos^2 x$
13. $h(t) = t^3 - 3^t \Rightarrow h'(t) = 3t^2 - 3^t \ln 3$ [by Formula 5]
14. $y = 3 \cot(n\theta) \Rightarrow y' = 3[-\csc^2(n\theta) \cdot n] = -3n \csc^2(n\theta)$
15. $y = xe^{-x^2} \Rightarrow y' = xe^{-x^2}(-2x) + e^{-x^2} \cdot 1 = e^{-x^2}(-2x^2 + 1) = e^{-x^2}(1 - 2x^2)$
16. $y = e^{-5x} \cos 3x \Rightarrow y' = e^{-5x}(-3 \sin 3x) + (\cos 3x)(-5e^{-5x}) = -e^{-5x}(3 \sin 3x + 5 \cos 3x)$