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3.5 Homework 1

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17.  $g(x) = (1 + 4x)^5(3 + x - x^2)^8 \Rightarrow$

$$\begin{aligned}g'(x) &= (1 + 4x)^5 \cdot 8(3 + x - x^2)^7(1 - 2x) + (3 + x - x^2)^8 \cdot 5(1 + 4x)^4 \cdot 4 \\&= 4(1 + 4x)^4(3 + x - x^2)^7[2(1 + 4x)(1 - 2x) + 5(3 + x - x^2)] \\&= 4(1 + 4x)^4(3 + x - x^2)^7[(2 + 4x - 16x^2) + (15 + 5x - 5x^2)] = 4(1 + 4x)^4(3 + x - x^2)^7(17 + 9x - 21x^2)\end{aligned}$$

18.  $h(t) = (t^4 - 1)^3(t^3 + 1)^4 \Rightarrow$

$$\begin{aligned}h'(t) &= (t^4 - 1)^3 \cdot 4(t^3 + 1)^3(3t^2) + (t^3 + 1)^4 \cdot 3(t^4 - 1)^2(4t^3) \\&= 12t^2(t^4 - 1)^2(t^3 + 1)^3[(t^4 - 1) + t(t^3 + 1)] = 12t^2(t^4 - 1)^2(t^3 + 1)^3(2t^4 + t - 1)\end{aligned}$$

19.  $y = e^{x \cos x} \Rightarrow y' = e^{x \cos x} \cdot \frac{d}{dx}(x \cos x) = e^{x \cos x} [x(-\sin x) + (\cos x) \cdot 1] = e^{x \cos x}(\cos x - x \sin x)$

20. Using Formula 5 and the Chain Rule,  $y = 10^{1-x^2} \Rightarrow y' = 10^{1-x^2}(\ln 10) \cdot \frac{d}{dx}(1 - x^2) = -2x(\ln 10)10^{1-x^2}$ .

21.  $F(z) = \sqrt{\frac{z-1}{z+1}} = \left(\frac{z-1}{z+1}\right)^{1/2} \Rightarrow$

$$\begin{aligned}F'(z) &= \frac{1}{2}\left(\frac{z-1}{z+1}\right)^{-1/2} \cdot \frac{d}{dz}\left(\frac{z-1}{z+1}\right) = \frac{1}{2}\left(\frac{z+1}{z-1}\right)^{1/2} \cdot \frac{(z+1)(1) - (z-1)(1)}{(z+1)^2} \\&= \frac{1}{2}\frac{(z+1)^{1/2}}{(z-1)^{1/2}} \cdot \frac{z+1-z+1}{(z+1)^2} = \frac{1}{2}\frac{(z+1)^{1/2}}{(z-1)^{1/2}} \cdot \frac{2}{(z+1)^2} = \frac{1}{(z-1)^{1/2}(z+1)^{3/2}}\end{aligned}$$

22.  $G(y) = \left(\frac{y^2}{y+1}\right)^5 \Rightarrow G'(y) = 5\left(\frac{y^2}{y+1}\right)^4 \cdot \frac{(y+1)(2y) - y^2(1)}{(y+1)^2} = 5 \cdot \frac{y^8}{(y+1)^4} \cdot \frac{y(2y+2-y)}{(y+1)^2} = \frac{5y^9(y+2)}{(y+1)^6}$

23.  $y = \sec^2 x + \tan^2 x = (\sec x)^2 + (\tan x)^2 \Rightarrow$

$$y' = 2(\sec x)(\sec x \tan x) + 2(\tan x)(\sec^2 x) = 2\sec^2 x \tan x + 2\sec^2 x \tan x = 4\sec^2 x \tan x$$

24.  $y = e^{k \tan \sqrt{x}} \Rightarrow y' = e^{k \tan \sqrt{x}} \cdot \frac{d}{dx}(k \tan \sqrt{x}) = e^{k \tan \sqrt{x}} \left(k \sec^2 \sqrt{x} \cdot \frac{1}{2}x^{-1/2}\right) = \frac{k \sec^2 \sqrt{x}}{2\sqrt{x}} e^{k \tan \sqrt{x}}$

25.  $y = \frac{r}{\sqrt{r^2+1}} \Rightarrow$

$$\begin{aligned}y' &= \frac{\sqrt{r^2+1}(1) - r \cdot \frac{1}{2}(r^2+1)^{-1/2}(2r)}{(\sqrt{r^2+1})^2} = \frac{\sqrt{r^2+1} - \frac{r^2}{\sqrt{r^2+1}}}{(\sqrt{r^2+1})^2} = \frac{\frac{\sqrt{r^2+1}\sqrt{r^2+1} - r^2}{\sqrt{r^2+1}}}{(\sqrt{r^2+1})^2} \\&= \frac{(r^2+1) - r^2}{(\sqrt{r^2+1})^3} = \frac{1}{(r^2+1)^{3/2}} \text{ or } (r^2+1)^{-3/2}\end{aligned}$$

*Another solution:* Write  $y$  as a product and make use of the Product Rule.  $y = r(r^2+1)^{-1/2} \Rightarrow$

$$y' = r \cdot -\frac{1}{2}(r^2+1)^{-3/2}(2r) + (r^2+1)^{-1/2} \cdot 1 = (r^2+1)^{-3/2}[-r^2 + (r^2+1)] = (r^2+1)^{-3/2}(1) = (r^2+1)^{-3/2}.$$

The step that students usually have trouble with is factoring out  $(r^2+1)^{-3/2}$ . But this is no different than factoring out  $x^2$  from  $x^2 + x^5$ ; that is, we are just factoring out a factor with the *smallest* exponent that appears on it. In this case,  $-\frac{3}{2}$  is smaller than  $-\frac{1}{2}$ .

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26.  $y = \frac{e^u - e^{-u}}{e^u + e^{-u}} \Rightarrow$

$$y' = \frac{(e^u + e^{-u})(e^u - (-e^{-u})) - (e^u - e^{-u})(e^u + (-e^{-u}))}{(e^u + e^{-u})^2}$$

$$= \frac{e^{2u} + e^0 + e^0 + e^{-2u} - (e^{2u} - e^0 - e^0 + e^{-2u})}{(e^u + e^{-u})^2} = \frac{4e^0}{(e^u + e^{-u})^2} = \frac{4}{(e^u + e^{-u})^2}$$

27. Using Formula 5 and the Chain Rule,  $y = 2^{\sin \pi x} \Rightarrow$

$$y' = 2^{\sin \pi x} (\ln 2) \cdot \frac{d}{dx} (\sin \pi x) = 2^{\sin \pi x} (\ln 2) \cdot \cos \pi x \cdot \pi = 2^{\sin \pi x} (\pi \ln 2) \cos \pi x$$

28.  $y = \tan^2(3\theta) = (\tan 3\theta)^2 \Rightarrow y' = 2(\tan 3\theta) \cdot \frac{d}{d\theta} (\tan 3\theta) = 2 \tan 3\theta \cdot \sec^2 3\theta \cdot 3 = 6 \tan 3\theta \sec^2 3\theta$

29.  $y = \cot^2(\sin \theta) = [\cot(\sin \theta)]^2 \Rightarrow$

$$y' = 2[\cot(\sin \theta)] \cdot \frac{d}{d\theta} [\cot(\sin \theta)] = 2 \cot(\sin \theta) \cdot [-\csc^2(\sin \theta) \cdot \cos \theta] = -2 \cos \theta \cot(\sin \theta) \csc^2(\sin \theta)$$

30.  $y = \sin(\sin(\sin x)) \Rightarrow y' = \cos(\sin(\sin x)) \frac{d}{dx} (\sin(\sin x)) = \cos(\sin(\sin x)) \cos(\sin x) \cos x$

31.  $y = \sin(\tan \sqrt{\sin x}) \Rightarrow$

$$y' = \cos(\tan \sqrt{\sin x}) \cdot \frac{d}{dx} (\tan \sqrt{\sin x}) = \cos(\tan \sqrt{\sin x}) \sec^2 \sqrt{\sin x} \cdot \frac{d}{dx} (\sin x)^{1/2}$$

$$= \cos(\tan \sqrt{\sin x}) \sec^2 \sqrt{\sin x} \cdot \frac{1}{2} (\sin x)^{-1/2} \cdot \cos x = \cos(\tan \sqrt{\sin x}) \left( \sec^2 \sqrt{\sin x} \right) \left( \frac{1}{2 \sqrt{\sin x}} \right) (\cos x)$$

37. (a)  $y = \frac{2}{1 + e^{-x}} \Rightarrow y' = \frac{(1 + e^{-x})(0) - 2(-e^{-x})}{(1 + e^{-x})^2} = \frac{2e^{-x}}{(1 + e^{-x})^2}$  (b)

At  $(0, 1)$ ,  $y' = \frac{2e^0}{(1 + e^0)^2} = \frac{2(1)}{(1 + 1)^2} = \frac{2}{2^2} = \frac{1}{2}$ . So an equation of the tangent line is  $y - 1 = \frac{1}{2}(x - 0)$  or  $y = \frac{1}{2}x + 1$ .

