
Section 3.4 In Class Problems

3. $g(t) = t^3 \cos t \Rightarrow g'(t) = t^3(-\sin t) + (\cos t) \cdot 3t^2 = 3t^2 \cos t - t^3 \sin t$ or $t^2(3 \cos t - t \sin t)$

4. $f(x) = \sqrt{x} \sin x \Rightarrow f'(x) = \sqrt{x} \cos x + \sin x \left(\frac{1}{2}x^{-1/2}\right) = \sqrt{x} \cos x + \frac{\sin x}{2\sqrt{x}}$

5. $y = \sec \theta \tan \theta \Rightarrow y' = \sec \theta (\sec^2 \theta) + \tan \theta (\sec \theta \tan \theta) = \sec \theta (\sec^2 \theta + \tan^2 \theta)$. Using the identity $1 + \tan^2 \theta = \sec^2 \theta$, we can write alternative forms of the answer as $\sec \theta (1 + 2 \tan^2 \theta)$ or $\sec \theta (2 \sec^2 \theta - 1)$.

6. $y = e^u (\cos u + cu) \Rightarrow y' = e^u (-\sin u + c) + (\cos u + cu)e^u = e^u (\cos u - \sin u + cu + c)$

7. $y = \frac{x}{\cos x} \Rightarrow y' = \frac{(\cos x)(1) - (x)(-\sin x)}{(\cos x)^2} = \frac{\cos x + x \sin x}{\cos^2 x}$

8. $y = \frac{1 + \sin x}{x + \cos x} \Rightarrow$
$$y' = \frac{(x + \cos x)(\cos x) - (1 + \sin x)(1 - \sin x)}{(x + \cos x)^2} = \frac{x \cos x + \cos^2 x - (1 - \sin^2 x)}{(x + \cos x)^2}$$
$$= \frac{x \cos x + \cos^2 x - (\cos^2 x)}{(x + \cos x)^2} = \frac{x \cos x}{(x + \cos x)^2}$$

9. $f(\theta) = \frac{\sec \theta}{1 + \sec \theta} \Rightarrow$
$$f'(\theta) = \frac{(1 + \sec \theta)(\sec \theta \tan \theta) - (\sec \theta)(\sec \theta \tan \theta)}{(1 + \sec \theta)^2} = \frac{(\sec \theta \tan \theta)[(1 + \sec \theta) - \sec \theta]}{(1 + \sec \theta)^2} = \frac{\sec \theta \tan \theta}{(1 + \sec \theta)^2}$$

10. $y = \frac{1 - \sec x}{\tan x} \Rightarrow$
$$y' = \frac{\tan x (-\sec x \tan x) - (1 - \sec x)(\sec^2 x)}{(\tan x)^2} = \frac{\sec x (-\tan^2 x - \sec x + \sec^2 x)}{\tan^2 x} = \frac{\sec x (1 - \sec x)}{\tan^2 x}$$

11. Using Exercise 3.2.45(a), $f(x) = xe^x \csc x \Rightarrow$
$$f'(x) = (x)'e^x \csc x + x(e^x)' \csc x + xe^x (\csc x)' = 1e^x \csc x + xe^x \csc x + xe^x (-\cot x \csc x)$$
$$= e^x \csc x (1 + x - x \cot x)$$

12. $y = \csc \theta (\theta + \cot \theta) \Rightarrow$
$$y' = \csc \theta (1 - \csc^2 \theta) + (\theta + \cot \theta)(-\csc \theta \cot \theta) = \csc \theta (1 - \csc^2 \theta - \theta \cot \theta - \cot^2 \theta)$$
$$= \csc \theta (-\cot^2 \theta - \theta \cot \theta - \cot^2 \theta) \quad [1 + \cot^2 \theta = \csc^2 \theta]$$
$$= \csc \theta (-\theta \cot \theta - 2 \cot^2 \theta) = -\csc \theta \cot \theta (\theta + 2 \cot \theta)$$

13. $\frac{d}{dx} (\csc x) = \frac{d}{dx} \left(\frac{1}{\sin x} \right) = \frac{(\sin x)(0) - 1(\cos x)}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} = -\csc x \cot x$