
3.2 In Class Problems

3. By the Product Rule, $f(x) = x^2 e^x \Rightarrow f'(x) = x^2 \frac{d}{dx}(e^x) + e^x \frac{d}{dx}(x^2) = x^2 e^x + e^x(2x) = x e^x(x + 2)$.

5. By the Quotient Rule, $y = \frac{e^x}{x^2} \Rightarrow$

$$y' = \frac{x^2 \frac{d}{dx}(e^x) - e^x \frac{d}{dx}(x^2)}{(x^2)^2} = \frac{x^2(e^x) - e^x(2x)}{x^4} = \frac{x e^x(x - 2)}{x^4} = \frac{e^x(x - 2)}{x^3}.$$

7. $g(x) = \frac{3x - 1}{2x + 1} \xrightarrow{\text{QR}} g'(x) = \frac{(2x + 1)(3) - (3x - 1)(2)}{(2x + 1)^2} = \frac{6x + 3 - 6x + 2}{(2x + 1)^2} = \frac{5}{(2x + 1)^2}$

9. $F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3) = (y^{-2} - 3y^{-4})(y + 5y^3) \xrightarrow{\text{PR}}$

$$\begin{aligned} F'(y) &= (y^{-2} - 3y^{-4})(1 + 15y^2) + (y + 5y^3)(-2y^{-3} + 12y^{-5}) \\ &= (y^{-2} + 15 - 3y^{-4} - 45y^{-2}) + (-2y^{-2} + 12y^{-4} - 10 + 60y^{-2}) \\ &= 5 + 14y^{-2} + 9y^{-4} \text{ or } 5 + 14/y^2 + 9/y^4 \end{aligned}$$

11. $y = \frac{t^2}{3t^2 - 2t + 1} \xrightarrow{\text{QR}}$

$$y' = \frac{(3t^2 - 2t + 1)(2t) - t^2(6t - 2)}{(3t^2 - 2t + 1)^2} = \frac{2t[3t^2 - 2t + 1 - t(3t - 1)]}{(3t^2 - 2t + 1)^2} = \frac{2t(3t^2 - 2t + 1 - 3t^2 + t)}{(3t^2 - 2t + 1)^2} = \frac{2t(1 - t)}{(3t^2 - 2t + 1)^2}$$

13. $y = (r^2 - 2r)e^r \xrightarrow{\text{PR}} y' = (r^2 - 2r)(e^r) + e^r(2r - 2) = e^r(r^2 - 2r + 2r - 2) = e^r(r^2 - 2)$

21. $y = 2xe^x \Rightarrow y' = 2(x \cdot e^x + e^x \cdot 1) = 2e^x((x + 1))$. At $(0, 0)$, $y' = 2e^0(0 + 1) = 2 \cdot 1 \cdot 1 = 2$, and an equation of the tangent line is $y - 0 = 2(x - 0)$, or $y = 2x$. The slope of the normal line is $-\frac{1}{2}$, so an equation of the normal line is $y - 0 = -\frac{1}{2}(x - 0)$, or $y = -\frac{1}{2}x$.

31. We are given that $f(5) = 1$, $f'(5) = 6$, $g(5) = -3$, and $g'(5) = 2$.

(a) $(fg)'(5) = f(5)g'(5) + g(5)f'(5) = (1)(2) + (-3)(6) = 2 - 18 = -16$

(b) $\left(\frac{f}{g}\right)'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{(-3)(6) - (1)(2)}{(-3)^2} = -\frac{20}{9}$

(c) $\left(\frac{g}{f}\right)'(5) = \frac{f(5)g'(5) - g(5)f'(5)}{[f(5)]^2} = \frac{(1)(2) - (-3)(6)}{(1)^2} = 20$

35. (a) From the graphs of f and g , we obtain the following values: $f(1) = 2$ since the point $(1, 2)$ is on the graph of f ;

$g(1) = 1$ since the point $(1, 1)$ is on the graph of g ; $f'(1) = 2$ since the slope of the line segment between $(0, 0)$ and $(2, 4)$

is $\frac{4 - 0}{2 - 0} = 2$; $g'(1) = -1$ since the slope of the line segment between $(-2, 4)$ and $(2, 0)$ is $\frac{0 - 4}{2 - (-2)} = -1$.

Now $u(x) = f(x)g(x)$, so $u'(1) = f(1)g'(1) + g(1)f'(1) = 2 \cdot (-1) + 1 \cdot 2 = 0$.

(b) $v(x) = f(x)/g(x)$, so $v'(5) = \frac{g(5)f'(5) - f(5)g'(5)}{[g(5)]^2} = \frac{2(-\frac{1}{3}) - 3 \cdot \frac{2}{3}}{2^2} = \frac{-\frac{8}{3}}{4} = -\frac{2}{3}$