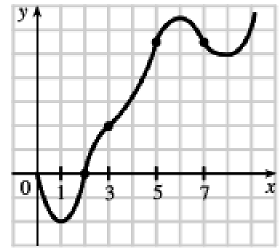
12. (a) f is increasing where f' is positive, on $(1, 6)$ and $(8, \infty)$, and decreasing where f' is negative, on $(0, 1)$ and $(6, 8)$.

(b) f has a local maximum where f' changes from positive to negative, at $x = 6$, and local minima where f' changes from negative to positive, at $x = 1$ and at $x = 8$.

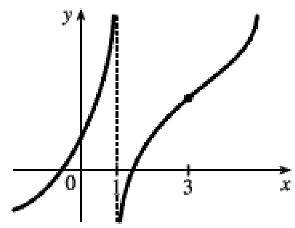
(c) f is concave upward where f' is increasing, that is, on $(0, 2)$, $(3, 5)$, and $(7, \infty)$, and concave downward where f' is decreasing, that is, on $(2, 3)$ and $(5, 7)$.

(d) There are points of inflection where f changes its direction of concavity, at $x = 2$, $x = 3$, $x = 5$ and $x = 7$.

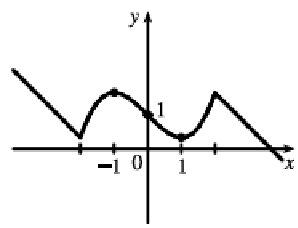
(e)



16. $f'(x) > 0$ for all $x \neq 1$ with vertical asymptote $x = 1$, so f is increasing on $(-\infty, 1)$ and $(1, \infty)$. $f''(x) > 0$ if $x < 1$ or $x > 3$, and $f''(x) < 0$ if $1 < x < 3$, so f is concave upward on $(-\infty, 1)$ and $(3, \infty)$, and concave downward on $(1, 3)$. There is an inflection point when $x = 3$.



18. $f'(1) = f'(-1) = 0 \quad \Rightarrow \quad$ horizontal tangents at $x = \pm 1$. $f'(x) < 0$ if $|x| < 1$ $\Rightarrow \quad f$ is decreasing on $(-1, 1)$. $f'(x) > 0$ if $1 < |x| < 2 \quad \Rightarrow \quad f$ is increasing on $(-2, -1)$ and $(1, 2)$. $f'(x) = -1$ if $|x| > 2 \quad \Rightarrow \quad$ the graph of f has constant slope -1 on $(-\infty, -2)$ and $(2, \infty)$. $f''(x) < 0$ if $-2 < x < 0 \quad \Rightarrow$ f is concave downward on $(-2, 0)$. The point $(0, 1)$ is an inflection point.



23. (a) To find the intervals on which f is increasing, we need to find the intervals on which $f'(x) = 3x^2 - 1$ is positive. $3x^2 - 1 > 0 \quad \Leftrightarrow \quad 3x^2 > 1 \quad \Leftrightarrow \quad x^2 > \frac{1}{3} \quad \Leftrightarrow \quad |x| > \sqrt{\frac{1}{3}}$, so $x \in \left(-\infty, -\sqrt{\frac{1}{3}}\right) \cup \left(\sqrt{\frac{1}{3}}, \infty\right)$. Thus, f is increasing on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and on $\left(\sqrt{\frac{1}{3}}, \infty\right)$. In a similar fashion, f is decreasing on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$.

(b) To find the intervals on which f is concave upward, we need to find the intervals on which $f''(x) = 6x$ is positive. $6x > 0 \quad \Leftrightarrow \quad x > 0$. So f is concave upward on $(0, \infty)$ and f is concave downward on $(-\infty, 0)$.

(c) There is an inflection point at $(0, 0)$ since f changes its direction of concavity at $x = 0$.

24. (a) $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{[(x+h)^4 - 2(x+h)^2] - (x^4 - 2x^2)}{h}$

$= \lim_{h\to 0} \frac{(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 2x^2 - 4xh - 2h^2) - (x^4 - 2x^2)}{h}$

$= \lim_{h\to 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4 - 4xh - 2h^2}{h} = \lim_{h\to 0} (4x^3 + 6x^2h + 4xh^2 + h^3 - 4x - 2h) = 4x^3 - 4x$

$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h\to 0} \frac{[4(x+h)^3 - 4(x+h)] - (4x^3 - 4x)}{h}$

$= \lim_{h\to 0} \frac{(4x^3 + 12x^2h + 12xh^2 + 4h^3 - 4x - 4h) - (4x^3 - 4x)}{h} = \lim_{h\to 0} \frac{12x^2h + 12xh^2 + 4h^3 - 4h}{h}$

$= \lim_{h\to 0} (12x^2 + 12xh + 4h^2 - 4) = 12x^2 - 4$

(b) $f'(x) > 0 \;\Leftrightarrow\; 4x^3 - 4x > 0 \;\Leftrightarrow\; 4x\left(x^2 - 1\right) > 0 \;\Leftrightarrow\; 4x(x+1)(x-1) > 0$, so f is increasing on $(-1, 0)$ and $(1, \infty)$ and f is decreasing on $(-\infty, -1)$ and $(0, 1)$.

(c) $f''(x) > 0 \;\Leftrightarrow\; 12x^2 - 4 > 0 \;\Leftrightarrow\; 12x^2 > 4 \;\Leftrightarrow\; x^2 > \frac{1}{3} \;\Leftrightarrow\; |x| > \sqrt{\frac{1}{3}}$, so f is CU on $\left(-\infty, -\sqrt{\frac{1}{3}}\right)$ and $\left(\sqrt{\frac{1}{3}}, \infty\right)$ and f is CD on $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$.

26. We know right away that c cannot be f's antiderivative, since the slope of c is not zero at the x-value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f.