31. f is not differentiable at $x = -4$, because the graph has a corner there, and at $x = 0$, because there is a discontinuity there.

32. f is not differentiable at $x = 0$, because there is a discontinuity there, and at $x = 3$, because the graph has a vertical tangent there.

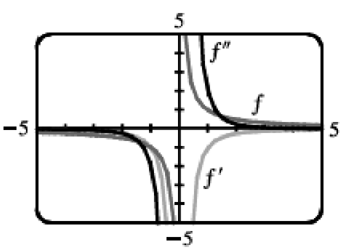
33. f is not differentiable at $x = -1$, because the graph has a vertical tangent there, and at $x = 4$, because the graph has a corner there.

34. f is not differentiable at $x = -1$, because there is a discontinuity there, and at $x = 2$, because the graph has a corner there.

38. Where d has horizontal tangents, only c is 0, so $d' = c$. c has negative tangents for $x < 0$ and b is the only graph that is negative for $x < 0$, so $c' = b$. b has positive tangents on $\mathbb{R}$ (except at $x = 0$), and the only graph that is positive on the same domain is a, so $b' = a$. We conclude that $d = f$, $c = f'$, $b = f''$, and $a = f'''$.

42. $$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h\to 0} \frac{x - (x+h)}{hx(x+h)} = \lim_{h\to 0} \frac{-h}{hx(x+h)} = \lim_{h\to 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}$$

$$f''(x) = \lim_{h\to 0} \frac{f'(x+h) - f'(x)}{h} = \lim_{h\to 0} \frac{-\frac{1}{(x+h)^2} - \left(-\frac{1}{x^2}\right)}{h} = \lim_{h\to 0} \frac{-x^2 + (x+h)^2}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{2hx + h^2}{hx^2(x+h)^2}$$
$$= \lim_{h\to 0} \frac{2x + h}{x^2(x+h)^2} = \frac{2x}{x^4} = \frac{2}{x^3}$$



We see from the graph that our answers are reasonable because the graph of f' is that of an even function and is negative for all $x \neq 0$, and the graph of f'' is that of an odd function (negative for $x < 0$ and positive for $x > 0$).

47. $$f(x) = |x - 6| = \begin{cases} x - 6 & \text{if } x - 6 \geq 6 \\ -(x-6) & \text{if } x - 6 < 0 \end{cases} = \begin{cases} x - 6 & \text{if } x \geq 6 \\ 6 - x & \text{if } x < 6 \end{cases}$$

So the right-hand limit is $\lim\limits_{x\to 6^+} \frac{f(x) - f(6)}{x - 6} = \lim\limits_{x\to 6^+} \frac{|x-6| - 0}{x - 6} = \lim\limits_{x\to 6^+} \frac{x-6}{x-6} = \lim\limits_{x\to 6^+} 1 = 1$, and the left-hand limit is $\lim\limits_{x\to 6^-} \frac{f(x) - f(6)}{x - 6} = \lim\limits_{x\to 6^-} \frac{|x-6| - 0}{x - 6} = \lim\limits_{x\to 6^-} \frac{6-x}{x-6} = \lim\limits_{x\to 6^-} (-1) = -1$. Since these limits are not equal, $f'(6) = \lim\limits_{x\to 6} \frac{f(x) - f(6)}{x - 6}$ does not exist and f is not differentiable at 6.

However, a formula for f' is $f'(x) = \begin{cases} 1 & \text{if } x > 6 \\ -1 & \text{if } x < 6 \end{cases}$

Another way of writing the formula is $f'(x) = \frac{x-6}{|x-6|}$.