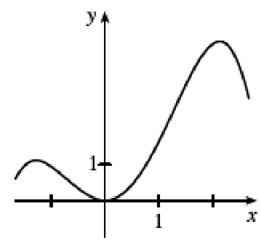
6. We begin by drawing a curve through the origin with a slope of 0 to satisfy $g(0) = 0$ and $g'(0) = 0$. The curve should have a slope of -1, 3, and 1 as we pass over $x = -1, 1$, and 2, respectively.

Note: In the figure, $y' = 0$ when $x \approx -1.27$ or 2.13.



8. Using Definition 2 with $g(x) = 1 - x^3$ and the point $(0, 1)$, we have

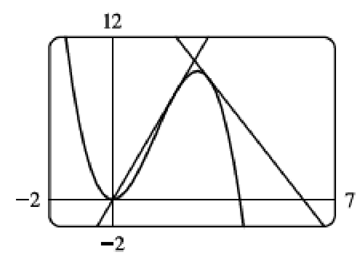
$$g'(0) = \lim_{h\to 0} \frac{g(0+h) - g(0)}{h} = \lim_{h\to 0} \frac{\left[1 - (0+h)^3\right] - 1}{h} = \lim_{h\to 0} \frac{(1-h^3) - 1}{h} = \lim_{h\to 0}(-h^2) = 0$$

So an equation of the tangent line is $y - 1 = 0(x - 0)$ or $y = 1$.

10. (a) Using Definition 2 with $G(x) = 4x^2 - x^3$, we have

$$\begin{aligned} G'(a) &= \lim_{h\to 0} \frac{G(a+h) - G(a)}{h} = \lim_{h\to 0} \frac{[4(a+h)^2 - (a+h)^3] - (4a^2 - a^3)}{h} \\ &= \lim_{h\to 0} \frac{4a^2 + 8ah + 4h^2 - (a^3 + 3a^2h + 3ah^2 + h^3) - 4a^2 + a^3}{h} = \lim_{h\to 0} \frac{8ah + 4h^2 - 3a^2h - 3ah^2 - h^3}{h} \\ &= \lim_{h\to 0} \frac{h(8a + 4h - 3a^2 - 3ah - h^{2)}}{h} = \lim_{h\to 0}(8a + 4h - 3a^2 - 3ah - h^2) = 8a - 3a^2 \end{aligned}$$

(b) At the point $(2, 8)$, $G'(2) = 16 - 12 = 4$, and an equation of the tangent line is $y - 8 = 4(x - 2)$, or $y = 4x$. At the point $(3, 9)$, $G'(3) = 24 - 27 = -3$, and an equation of the tangent line is $y - 9 = -3(x - 3)$, or $y = -3x + 18$.



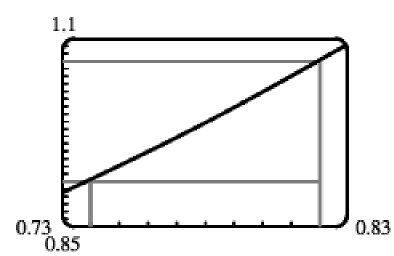
12. (a) $g'\left(\frac{\pi}{4}\right) = \lim_{h\to 0} \frac{g\left(\frac{\pi}{4} + h\right) - g\left(\frac{\pi}{4}\right)}{h} = \lim_{h\to 0} \frac{\tan\left(\frac{\pi}{4} + h\right) - \tan\left(\frac{\pi}{4}\right)}{h}$

So let $G(h) = \dfrac{\tan\left(\frac{\pi}{4} + h\right) - 1}{h}$. We calculate:

h	$G(h)$	h	$G(h)$
0.1	2.2305	-0.1	1.8237
0.01	2.0203	-0.01	1.9803
0.001	2.0020	-0.001	1.9980
0.0001	2.0002	-0.0001	1.9998

We estimate that $g'\left(\frac{\pi}{4}\right) = 2$.

(b)



From the graph, we estimate that the slope of the tangent is about $\dfrac{1.07 - 0.91}{0.82 - 0.74} = \dfrac{0.16}{0.08} = 2$.

15. Use Definition 2 with $f(t) = (2t+1)/(t+3)$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{2(a+h)+1}{(a+h)+3} - \dfrac{2a+1}{a+3}}{h} = \lim_{h\to 0} \frac{(2a+2h+1)(a+3)-(2a+1)(a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h\to 0} \frac{(2a^2+6a+2ah+6h+a+3)-(2a^2+2ah+6a+a+h+3)}{h(a+h+3)(a+3)}$$

$$= \lim_{h\to 0} \frac{5h}{h(a+h+3)(a+3)} = \lim_{h\to 0} \frac{5}{(a+h+3)(a+3)} = \frac{5}{(a+3)^2}$$

17. Use Definition 2 with $f(x) = 1/\sqrt{x+2}$.

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\dfrac{1}{\sqrt{(a+h)+2}} - \dfrac{1}{\sqrt{a+2}}}{h} = \lim_{h\to 0} \frac{\dfrac{\sqrt{a+2}-\sqrt{a+h+2}}{\sqrt{a+h+2}\,\sqrt{a+2}}}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sqrt{a+2}-\sqrt{a+h+2}}{h\,\sqrt{a+h+2}\,\sqrt{a+2}} \cdot \frac{\sqrt{a+2}+\sqrt{a+h+2}}{\sqrt{a+2}+\sqrt{a+h+2}}\right] = \lim_{h\to 0} \frac{(a+2)-(a+h+2)}{h\sqrt{a+h+2}\,\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+h+2}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{h\sqrt{a+h+2}\,\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+h+2}\right)} = \lim_{h\to 0} \frac{-1}{\sqrt{a+h+2}\,\sqrt{a+2}\left(\sqrt{a+2}+\sqrt{a+h+2}\right)}$$

$$= \frac{-1}{\left(\sqrt{a+2}\right)^2\left(2\sqrt{a+2}\right)} = -\frac{1}{2(a+2)^{3/2}}$$

Note that the answers to Exercises 19 – 24 are not unique.

19. By Definition 2, $\lim\limits_{h\to 0} \dfrac{(1+h)^{10}-1}{h} = f'(1)$, where $f(x) = x^{10}$ and $a = 1$.

Or: By Definition 2, $\lim\limits_{h\to 0} \dfrac{(1+h)^{10}-1}{h} = f'(0)$, where $f(x) = (1+x)^{10}$ and $a = 0$.

Note that the answers to Exercises 19 – 24 are not unique.

20. By Definition 2, $\lim\limits_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h} = f'(16)$, where $f(x) = \sqrt[4]{x}$ and $a = 16$.

Or: By Definition 2, $\lim\limits_{h\to 0} \dfrac{\sqrt[4]{16+h}-2}{h} = f'(0)$, where $f(x) = \sqrt[4]{16+x}$ and $a = 0$.

27. (a) $f'(x)$ is the rate of change of the production cost with respect to the number of ounces of gold produced. Its units are dollars per ounce.

(b) After 800 ounces of gold have been produced, the rate at which the production cost is increasing is \$17/ounce. So the cost of producing the 800th (or 801st) ounce is about \$17.

(c) In the short term, the values of $f'(x)$ will decrease because more efficient use is made of start-up costs as x increases. But eventually $f'(x)$ might increase due to large-scale operations.