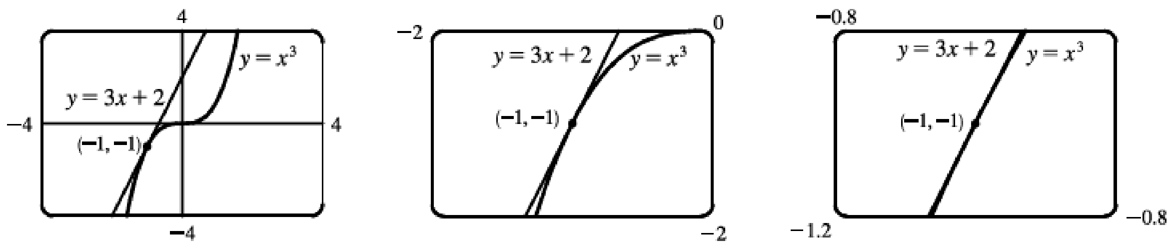
6. (a) (i) $m = \lim_{x\to -1} \dfrac{f(x) - f(-1)}{x - (-1)} = \lim_{x\to -1} \dfrac{x^3 - (-1)}{x+1} = \lim_{x\to -1} \dfrac{(x+1)(x^2 - x + 1)}{x+1} = \lim_{x\to -1} (x^2 - x + 1) = 3$

(ii) $m = \lim_{h\to 0} \dfrac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \dfrac{(-1+h)^3 - (-1)}{h} = \lim_{h\to 0} \dfrac{h^3 - 3h^2 + 3h - 1 + 1}{h}$

$= \lim_{h\to 0} (h^2 - 3h + 3) = 3$

(b) $y - (-1) = 3\,[x - (-1)] \quad \Leftrightarrow \quad y + 1 = 3x + 3 \quad \Leftrightarrow \quad y = 3x + 2$

(c)



8. Using (1),

$$m = \lim_{x\to -1} \frac{(2x^3 - 5x) - 3}{x - (-1)} = \lim_{x\to -1} \frac{2x^3 - 5x - 3}{x+1} = \lim_{x\to -1} \frac{(2x^2 - 2x - 3)(x+1)}{x+1} = \lim_{x\to -1} (2x^2 - 2x - 3) = 1.$$

Tangent line: $y - 3 = 1\,[x - (-1)] \quad \Leftrightarrow \quad y = x + 4$

9. Using (1), $m = \lim_{x\to 1} \dfrac{\sqrt{x} - \sqrt{1}}{x - 1} = \lim_{x\to 1} \dfrac{(\sqrt{x} - 1)(\sqrt{x} + 1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \dfrac{x - 1}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1} \dfrac{1}{\sqrt{x}+1} = \dfrac{1}{2}.$

Tangent line: $y - 1 = \frac{1}{2}(x - 1) \quad \Leftrightarrow \quad y = \frac{1}{2}x + \frac{1}{2}.$

15. (a) Since the slope of the tangent at $t = 0$ is 0, the car's initial velocity was 0.

(b) The slope of the tangent is greater at C than at B, so the car was going faster at C.

(c) Near A, the tangent lines are becoming steeper as x increases, so the velocity was increasing, so the car was speeding up. Near B, the tangent lines are becoming less steep, so the car was slowing down. The steepest tangent near C is the one at C, so at C the car had just finished speeding up, and was about to start slowing down.

(d) Between D and E, the slope of the tangent is 0, so the car did not move during that time.

18. (a) $v(1) = \lim_{h\to 0} \dfrac{H(1+h) - H(1)}{h} = \lim_{h\to 0} \dfrac{(58 + 58h - 0.83 - 1.66h - 0.83h^2) - 57.17}{h}$

$= \lim_{h\to 0} (56.34 - 0.83h) = 56.34 \text{ m/s}$

(b) $v(a) = \lim_{h\to 0} \dfrac{H(a+h) - H(a)}{h} = \lim_{h\to 0} \dfrac{(58a + 58h - 0.83a^2 - 1.66ah - 0.83h^2) - (58a - 0.83a^2)}{h}$

$= \lim_{h\to 0} (58 - 1.66a - 0.83h) = 58 - 1.66a \text{ m/s}$

(c) The arrow strikes the moon when the height is 0, that is, $58t - 0.83t^2 = 0 \quad \Leftrightarrow \quad t(58 - 0.83t) = 0 \quad \Leftrightarrow$

$t = \dfrac{58}{0.83} \approx 69.9$ s (since t can't be 0).

(d) Using the time from part (c), $v\left(\dfrac{58}{0.83}\right) = 58 - 1.66\left(\dfrac{58}{0.83}\right) = -58$ m/s. Thus, the arrow will have a velocity of -58 m/s.

20. (a) The average velocity between times t and $t+h$ is

$$\frac{s(t+h)-s(t)}{(t+h)-t} = \frac{(t+h)^2-8(t+h)+18-\left(t^2-8t+18\right)}{h} = \frac{t^2+2th+h^2-8t-8h+18-t^2+8t-18}{h}$$

$$= \frac{2th+h^2-8h}{h} = (2t+h-8) \ \ \text{m/s}.$$

(i) $[3,4]$: $t=3$, $h=4-3=1$, so the average velocity is $2(3)+1-8=-1$ m/s.

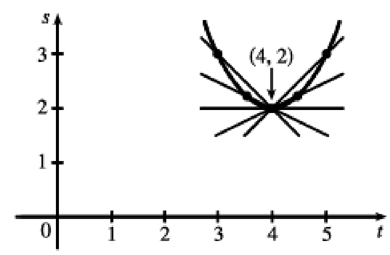
(ii) $[3.5,4]$: $t=3.5$, $h=0.5$, so the average velocity is $2(3.5)+0.5-8=-0.5$ m/s.

(iii) $[4,5]$: $t=4$, $h=1$, so the average velocity is $2(4)+1-8=1$ m/s.

(iv) $[4,4.5]$: $t=4$, $h=0.5$, so the average velocity is $2(4)+0.5-8=0.5$ m/s.

(b) $v(t)=\lim\limits_{h\to 0}\dfrac{s(t+h)-s(t)}{h}=\lim\limits_{h\to 0}(2t+h-8)=2t-8$, so $v(4)=0$.

(c)



25. (a) (i) $[2000,2002]$: $\dfrac{P(2002)-P(2000)}{2002-2000}=\dfrac{77-55}{2}=\dfrac{22}{2}=11$ percent/year

(ii) $[2000,2001]$: $\dfrac{P(2001)-P(2000)}{2001-2000}=\dfrac{68-55}{1}=13$ percent/year

(iii) $[1999,2000]$: $\dfrac{P(2000)-P(1999)}{2000-1999}=\dfrac{55-39}{1}=16$ percent/year

(b) Using the values from (ii) and (iii), we have $\dfrac{13+16}{2}=14.5$ percent/year.

(c) Estimating A as $(1999,40)$ and B as $(2001,70)$, the slope at 2000 is

$$\frac{70-40}{2001-1999}=\frac{30}{2}=15 \text{ percent/year.}$$