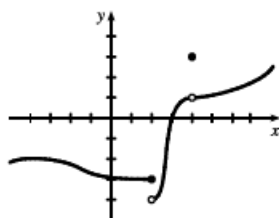


4.  $g$  is continuous on  $[-4, -2)$ ,  $(-2, 2)$ ,  $[2, 4)$ ,  $(4, 6)$ , and  $(6, 8)$ .

6.



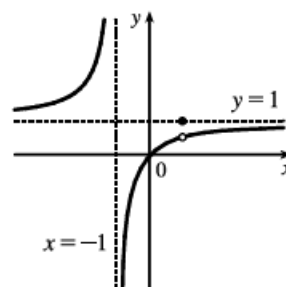
$$10. \lim_{x \rightarrow 4} f(x) = \lim_{x \rightarrow 4} (x^2 + \sqrt{7-x}) = \lim_{x \rightarrow 4} x^2 + \sqrt{\lim_{x \rightarrow 4} 7 - \lim_{x \rightarrow 4} x} = 4^2 + \sqrt{7-4} = 16 + \sqrt{3} = f(4).$$

By the definition of continuity,  $f$  is continuous at  $a = 4$ .

$$16. f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & \text{if } x \neq 1 \\ 1 & \text{if } x = 1 \end{cases}$$

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x^2 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x+1)(x-1)} = \lim_{x \rightarrow 1} \frac{x}{x+1} = \frac{1}{2},$$

but  $f(1) = 1$ , so  $f$  is discontinuous at 1.



19. By Theorem 5, the polynomials  $x^2$  and  $2x - 1$  are continuous on  $(-\infty, \infty)$ . By Theorem 7, the root function  $\sqrt{x}$  is continuous on  $[0, \infty)$ . By Theorem 9, the composite function  $\sqrt{2x-1}$  is continuous on its domain,  $[\frac{1}{2}, \infty)$ . By part 1 of Theorem 4, the sum  $R(x) = x^2 + \sqrt{2x-1}$  is continuous on  $[\frac{1}{2}, \infty)$ .

25. Because we are dealing with root functions,  $5 + \sqrt{x}$  is continuous on  $[0, \infty)$ ,  $\sqrt{x+5}$  is continuous on  $[-5, \infty)$ , so the quotient  $f(x) = \frac{5 + \sqrt{x}}{\sqrt{5+x}}$  is continuous on  $[0, \infty)$ . Since  $f$  is continuous at  $x = 4$ ,  $\lim_{x \rightarrow 4} f(x) = f(4) = \frac{7}{3}$ .

26. Because  $x$  is continuous on  $\mathbb{R}$ ,  $\sin x$  is continuous on  $\mathbb{R}$ , and  $x + \sin x$  is continuous on  $\mathbb{R}$ , the composite function  $f(x) = \sin(x + \sin x)$  is continuous on  $\mathbb{R}$ , so  $\lim_{x \rightarrow \pi} f(x) = f(\pi) = \sin(\pi + \sin \pi) = \sin \pi = 0$ .

$$29. f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$$

By Theorem 5, since  $f(x)$  equals the polynomial  $x^2$  on  $(-\infty, 1)$ ,  $f$  is continuous on  $(-\infty, 1)$ . By Theorem 7, since  $f(x)$  equals the root function  $\sqrt{x}$  on  $(1, \infty)$ ,  $f$  is continuous on  $(1, \infty)$ . At  $x = 1$ ,  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 = 1$  and

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} \sqrt{x} = 1$ . Thus,  $\lim_{x \rightarrow 1} f(x)$  exists and equals 1. Also,  $f(1) = \sqrt{1} = 1$ . Thus,  $f$  is continuous at  $x = 1$ . We conclude that  $f$  is continuous on  $(-\infty, \infty)$ .