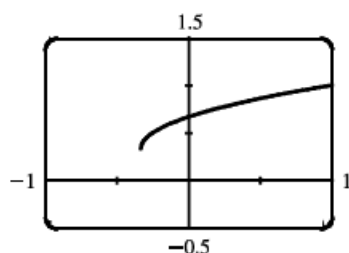

Sec. 2.3 In Class Problems

2. (a) $\lim_{x \rightarrow 2} [f(x) + g(x)] = \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x) = 2 + 0 = 2$
(b) $\lim_{x \rightarrow 1} g(x)$ does not exist since its left- and right-hand limits are not equal, so the given limit does not exist.
(c) $\lim_{x \rightarrow 0} [f(x)g(x)] = \lim_{x \rightarrow 0} f(x) \cdot \lim_{x \rightarrow 0} g(x) = 0 \cdot 1.3 = 0$
(d) Since $\lim_{x \rightarrow -1} g(x) = 0$ and g is in the denominator, but $\lim_{x \rightarrow -1} f(x) = -1 \neq 0$, the given limit does not exist.
(e) $\lim_{x \rightarrow 2} x^3 f(x) = \left[\lim_{x \rightarrow 2} x^3 \right] \left[\lim_{x \rightarrow 2} f(x) \right] = 2^3 \cdot 2 = 16$
(f) $\lim_{x \rightarrow 1} \sqrt{3 + f(x)} = \sqrt{3 + \lim_{x \rightarrow 1} f(x)} = \sqrt{3 + 1} = 2$
3. $\lim_{x \rightarrow -2} (3x^4 + 2x^2 - x + 1) = \lim_{x \rightarrow -2} 3x^4 + \lim_{x \rightarrow -2} 2x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1$ [Limit Laws 1 and 2]
 $= 3 \lim_{x \rightarrow -2} x^4 + 2 \lim_{x \rightarrow -2} x^2 - \lim_{x \rightarrow -2} x + \lim_{x \rightarrow -2} 1$ [3]
 $= 3(-2)^4 + 2(-2)^2 - (-2) + (1)$ [9, 8, and 7]
 $= 48 + 8 + 2 + 1 = 59$
6. $\lim_{u \rightarrow -2} \sqrt{u^4 + 3u + 6} = \sqrt{\lim_{u \rightarrow -2} (u^4 + 3u + 6)}$ [11]
 $= \sqrt{\lim_{u \rightarrow -2} u^4 + 3 \lim_{u \rightarrow -2} u + \lim_{u \rightarrow -2} 6}$ [1, 2, and 3]
 $= \sqrt{(-2)^4 + 3(-2) + 6}$ [9, 8, and 7]
 $= \sqrt{16 - 6 + 6} = \sqrt{16} = 4$
8. (a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.
(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \rightarrow 2$, just as in Example 3. Remember that in finding $\lim_{x \rightarrow a} f(x)$, we never consider $x = a$.
13. $\lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3} = \lim_{t \rightarrow -3} \frac{(t+3)(t-3)}{(2t+1)(t+3)} = \lim_{t \rightarrow -3} \frac{t-3}{2t+1} = \frac{-3-3}{2(-3)+1} = \frac{-6}{-5} = \frac{6}{5}$
17. By the formula for the sum of cubes, we have
 $\lim_{x \rightarrow -2} \frac{x+2}{x^3+8} = \lim_{x \rightarrow -2} \frac{x+2}{(x+2)(x^2-2x+4)} = \lim_{x \rightarrow -2} \frac{1}{x^2-2x+4} = \frac{1}{4+4+4} = \frac{1}{12}$

23. (a)



$$\lim_{x \rightarrow 0} \frac{x}{\sqrt{1+3x}-1} \approx \frac{2}{3}$$

(b)

x	$f(x)$
-0.001	0.6661663
-0.0001	0.6666167
-0.00001	0.6666617
-0.000001	0.6666662
0.000001	0.6666672
0.00001	0.6666717
0.0001	0.6667167
0.001	0.6671663

The limit appears to be $\frac{2}{3}$.

$$\begin{aligned}
 \text{(c) } \lim_{x \rightarrow 0} \left(\frac{x}{\sqrt{1+3x}-1} \cdot \frac{\sqrt{1+3x}+1}{\sqrt{1+3x}+1} \right) &= \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{(1+3x)-1} = \lim_{x \rightarrow 0} \frac{x(\sqrt{1+3x}+1)}{3x} \\
 &= \frac{1}{3} \lim_{x \rightarrow 0} (\sqrt{1+3x}+1) && \text{[Limit Law 3]} \\
 &= \frac{1}{3} \left[\sqrt{\lim_{x \rightarrow 0} (1+3x)} + \lim_{x \rightarrow 0} 1 \right] && \text{[1 and 11]} \\
 &= \frac{1}{3} \left(\sqrt{\lim_{x \rightarrow 0} 1 + 3 \lim_{x \rightarrow 0} x} + 1 \right) && \text{[1, 3, and 7]} \\
 &= \frac{1}{3} (\sqrt{1+3 \cdot 0} + 1) && \text{[7 and 8]} \\
 &= \frac{1}{3} (1+1) = \frac{2}{3}
 \end{aligned}$$