19. $\lim\limits_{x\to 7}\dfrac{\sqrt{x+2}-3}{x-7}=\lim\limits_{x\to 7}\dfrac{\sqrt{x+2}-3}{x-7}\cdot\dfrac{\sqrt{x+2}+3}{\sqrt{x+2}+3}=\lim\limits_{x\to 7}\dfrac{(x+2)-9}{(x-7)\left(\sqrt{x+2}+3\right)}$

$$=\lim_{x\to 7}\frac{x-7}{(x-7)\left(\sqrt{x+2}+3\right)}=\lim_{x\to 7}\frac{1}{\sqrt{x+2}+3}=\frac{1}{\sqrt{9}+3}=\frac{1}{6}$$

20. $\lim\limits_{h\to 0}\dfrac{(3+h)^{-1}-3^{-1}}{h}=\lim\limits_{h\to 0}\dfrac{\dfrac{1}{3+h}-\dfrac{1}{3}}{h}=\lim\limits_{h\to 0}\dfrac{3-(3+h)}{h(3+h)3}=\lim\limits_{h\to 0}\dfrac{-h}{h(3+h)3}$

$$=\lim_{h\to 0}\left[-\frac{1}{3(3+h)}\right]=-\frac{1}{\lim\limits_{h\to 0}[3(3+h)]}=-\frac{1}{3(3+0)}=-\frac{1}{9}$$

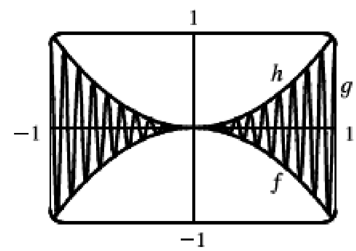
21. $\lim\limits_{x\to -4}\dfrac{\dfrac{1}{4}+\dfrac{1}{x}}{4+x}=\lim\limits_{x\to -4}\dfrac{\dfrac{x+4}{4x}}{4+x}=\lim\limits_{x\to -4}\dfrac{x+4}{4x(4+x)}=\lim\limits_{x\to -4}\dfrac{1}{4x}=\dfrac{1}{4(-4)}=-\dfrac{1}{16}$

25. Let $f(x)=-x^2$, $g(x)=x^2\cos 20\pi x$ and $h(x)=x^2$.

Then $-1\le\cos 20\pi x\le 1 \quad\Rightarrow\quad -x^2\le x^2\cos 20\pi x\le x^2 \quad\Rightarrow$

$f(x)\le g(x)\le h(x)$. So since $\lim\limits_{x\to 0}f(x)=\lim\limits_{x\to 0}h(x)=0$, by the Squeeze

Theorem we have $\lim\limits_{x\to 0}g(x)=0$.



27. We have $\lim\limits_{x\to 4}(4x-9)=4(4)-9=7$ and $\lim\limits_{x\to 4}\left(x^2-4x+7\right)=4^2-4(4)+7=7$. Since $4x-9\le f(x)\le x^2-4x+7$ for $x\ge 0$, $\lim\limits_{x\to 4}f(x)=7$ by the Squeeze Theorem.

35. (a) (i) If $x\to 1^+$, then $x>1$ and $g(x)=x-1$. Thus, $\lim\limits_{x\to 1^+}g(x)=\lim\limits_{x\to 1^+}(x-1)=1-1=0$.

(ii) If $x\to 1^-$, then $x<1$ and $g(x)=1-x^2$. Thus, $\lim\limits_{x\to 1^-}g(x)=\lim\limits_{x\to 1^-}(1-x^2)=1-1^2=0$.

Since the left- and right-hand limits of g at 1 are equal, $\lim\limits_{x\to 1}g(x)=0$.

(iii) If $x\to 0$, then $-1<x<1$ and $g(x)=1-x^2$. Thus, $\lim\limits_{x\to 0}g(x)=\lim\limits_{x\to 0}(1-x^2)=1-0^2=1$.

(iv) If $x\to -1^-$, then $x<-1$ and $g(x)=-x$. Thus, $\lim\limits_{x\to -1^-}g(x)=\lim\limits_{x\to -1^-}(-x)=-(-1)=1$.

(v) If $x\to -1^+$, then $-1<x<1$ and $g(x)=1-x^2$. Thus,

$$\lim_{x\to -1^+}g(x)=\lim_{x\to -1^+}(1-x^2)=1-(-1)^2=1-1=0$$

(vi) $\lim\limits_{x\to -1}g(x)$ does not exist because the limits in part (iv) and part (v) are not equal.

(b)